## Good Questions

# Great Ways to DIFFERENTIATE MATHEMATICS Instruction in the Standards-Based Classroom 

FOURTH EDITION

SAMPLE MATERIAL INSIDE

Marian Small Foreword by Carol Ann Tomlinson

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# Good Questions 

# Great Ways to Differentiate Mathematics Instruction in the Standards-Based Classroom 

Fourth Edition

## MARIAN SMALL

Foreword by Carol Ann Tomlinson

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The "My Own Questions and Tasks" worksheet (see Appendix B) is available at tcpress.com

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## Preface

FOR THE PAST 15 years, I have had the opportunity to develop a number of resources that support teachers who seek to differentiate instruction in mathematics. Much of the work I do focuses on differentiation in terms of student readiness and highlights two strategies: open questions and parallel tasks.

These strategies have been employed effectively by thousands of teachers throughout Canada and the United States, as well as overseas. What we have learned is that to differentiate instruction in math, it is important for teachers to think about "bigger ideas," as they do when teaching other subjects, rather than focus intensely on the usual very tight content pieces in mathematics that capture their attention.

## ORGANIZATION OF THE BOOK

The introductory chapter of this resource describes the rationale for differentiating math instruction and explains the two principal strategies that are employed throughout the book: open questions and parallel tasks. Nine content chapters built around the typical organization of mathematical content standards used by many states-as well as a chapter focusing on the mathematics of financial literacy-then illustrate applications of these strategies, followed by a final concluding chapter, an appendix listing the mathematical practices addressed throughout the content work, an appendix containing a template for teachers wishing to develop their own materials, a glossary, a bibliography, and an index.

Chapters 1-10 focus on the following content topics.

- Counting \& Cardinality / Number \& Operations in Base Ten
- Number \& Operations-Fractions (including decimals)
- The Number System
- Ratios \& Proportional Relationships
- Operations \& Algebraic Thinking
- Expressions \& Equations / Functions
- Financial Literacy
- Measurement \& Data
- Geometry
- Statistics \& Probability

The content strands are not developed sequentially, so Chapters 1-10 can be approached in any order. Within each of the content chapters, a set of big ideas is described that underlies the increasingly sophisticated teaching of that content throughout the grades. In this edition, each question or task is tagged to one or more of those big ideas, as well as to applicable standards for mathematical practice. Alternative versions of practice standards or process standards are used in various jurisdictions. Those applied here are listed in Appendix A.

The suggested differentiating questions and tasks are organized according to the primary, elementary, and middle-level grade bands set out in many states' standards. The object of differentiation is to teach the same broad concepts to students at different developmental levels. The multilayered organization of material within Chapters $1-10$ is intended to help make this possible.

Appendix B features a template worksheet that will assist teachers in developing their own materials in support of differentiated instruction through use of open questions and parallel tasks. An example of application of the worksheet appears in the Introduction.

The Glossary defines technical terms used throughout. Each word that appears in the Glossary is shown in boldface type at its first occurrence in the text, and each Glossary entry is annotated with the chapter and page number of the term's first occurrence.

The Bibliography highlights three types of resources: those that are referenced in other parts of the text, those that speak to the issues of teaching to big ideas and differentiating instruction, as well as a number of excellent sources for activities that can be used as-is or used as a starting point for creating open questions and parallel tasks.

The Index focuses on educational concepts-standards, student development, teaching methods and principles, and such-as opposed to mathematical concepts. To facilitate user access to the mathematical topics covered, an Index of Big Ideas is provided, listing all big ideas covered in the content chapters.

## MATHEMATICAL PRACTICE STANDARDS

The eight standards for mathematical practice that were addressed in the third edition are also addressed in this edition. These practice standards reflect "processes" that should be elicited from students and goals for orienting and developing students as mathematical thinkers as content is learned.

Some of these practices may or may not be brought out, depending on how a teacher handles the presented questions or tasks, but others are implicit no matter what direction the discussion of the question or task takes.

For example, almost all of the questions and tasks posed in this book require students to make sense of them and persevere in solving them. This is true with open questions, which are often deliberately vague and require students to make sense of the question before they can choose the direction in which to go. This is also true with parallel tasks, where students are likely to try to make sense of both options to decide with which one to proceed.

Students are frequently asked to explain their rationale, requiring them to construct viable arguments. Ideally, teachers would encourage students to critique each other's reasoning, although that would not be implicit in the question itself.

A great many of the questions provided require students to reason, either abstractly or, in the case of number or measurement questions, quantitatively.

Some questions bring out the other practices: modeling, using appropriate tools strategically, attending to precision, looking for and making use of structure, and/or looking for and expressing regularity in repeated reasoning. There are fewer questions attending to precision in this particular resource than in others precisely because differentiation must often allow for varying levels of precision.

Listed with each question and task are the practice standards that are clearly evoked in the question or task. This does not mean that teachers might not also evoke others, depending on their approach to student responses or the student responses themselves.

## CHANGES IN THE FOURTH EDITION

Readers familiar with the third edition of this book will note that there are about 60 new questions. The biggest change, though, is the inclusion of a new chapter on financial literacy. This is a topic formally addressed in only some states' standards, but is of value for all students.

Financial literacy is an interesting topic since there are both social issues and mathematical issues involved. Different jurisdictions may emphasize one over the other; this resource blends both, addressing essential understandings about both, although emphasizing the mathematics, since this is a mathematics resource. For example, one essential understanding focuses on different ways to show the same amount of money, while another focuses on the fact that financial decisions are based on considering many factors.

IT IS MY HOPE that teachers will embrace the two core strategies-open questions and parallel tasks-that are explained and demonstrated in this book, and find them, as I have, to be helpful to the many children who come into classrooms with highly differentiated mathematical preparation, skill, and confidence. Seeing a child who has been struggling in mathematics start to feel successful is an important goal in my teaching. I have seen the use of the strategies described in this volume make that happen over and over again.

