NELSON

TEACHER'S RESOURCE

7/8

Leaps and Bounds toward Math Understanding

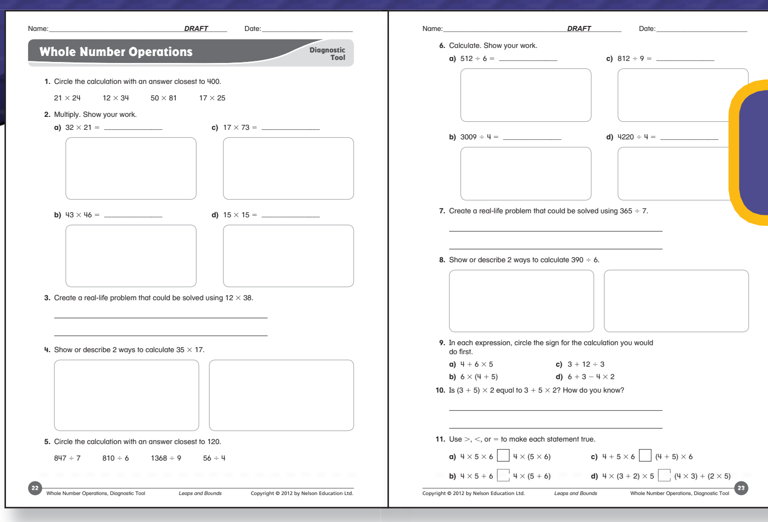
Marian Small

Teacher's Resource Sampler: Number

Topics 1 and 2, Draft Material

Leaps AND Bounds TOWARD Math Understanding

With Leaps and Bounds, mathematics intervention is as easy as 1, 2, 3!



Name: ______________ DRAFT   Date: ______________

Whole Number Operations

Diagnostic Tool

1. Circle the calculation with an answer closest to 400.

   21 × 24   12 × 34   50 × 81   17 × 25

2. Multiply. Show your work.

   a) 32 × 21 = ________   c) 17 × 73 = ________

   b) 43 × 46 = ________   d) 15 × 15 = ________

3. Create a real-life problem that could be solved using 12 × 38.

4. Show or describe 2 ways to calculate 35 × 17.

5. Circle the calculation with an answer closest to 120.

   847 ÷ 7   810 ÷ 6   1368 ÷ 9   56 ÷ 4

22 Whole Number Operations, Diagnostic Tool   *Leaps and Bounds*   Copyright © 2012 by Nelson Education Ltd.

Name: ______________ DRAFT   Date: ______________

6. Calculate. Show your work.

   a) 512 ÷ 6 = ________   c) 812 ÷ 9 = ________

   b) 3009 ÷ 4 = ________   d) 4220 ÷ 4 = ________

7. Create a real-life problem that could be solved using 365 ÷ 7.

8. Show or describe 2 ways to calculate 390 ÷ 6.

9. In each expression, circle the sign for the calculation you would do first.

   a) 4 + 6 × 5   c) 3 + 12 ÷ 3

   b) 6 × (4 + 5)   d) 6 + 3 − 4 × 2

10. Is (3 + 5) × 2 equal to 3 + 5 × 2? How do you know?

11. Use >, <, or = to make each statement true.

    a) 4 × 5 × 6 ☐ 4 × (5 × 6)   c) 4 + 5 × 6 ☐ (4 + 5) × 6

    b) 4 × 5 + 6 ☐ 4 × (5 + 6)   d) 4 × (3 + 2) × 5 ☐ (4 × 3) + (2 × 5)

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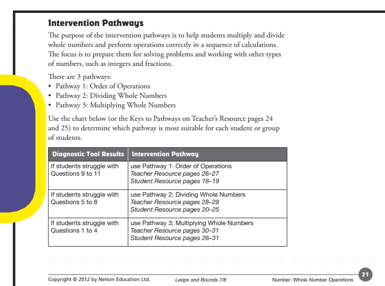
Step 1: Administer the diagnostic assessment

Teacher's Resource



Intervention Pathways

The purpose of the intervention pathways is to help students multiply and divide whole numbers and perform operations correctly in a sequence of calculations. The focus is to prepare them for solving problems and working with other types of numbers, such as integers and fractions.

There are 3 pathways:

- Pathway 1: Order of Operations
- Pathway 2: Dividing Whole Numbers
- Pathway 3: Multiplying Whole Numbers

Use the chart below (or the Keys to Pathways on Teacher's Resource pages 24 and 25) to determine which pathway is most suitable for each student or group of students.

Diagnostic Tool Results	Intervention Pathway
If students struggle with Questions 9 to 11	use Pathway 1: Order of Operations *Teacher Resource pages 26–27* *Student Resource pages 16–19*
If students struggle with Questions 5 to 8	use Pathway 2: Dividing Whole Numbers *Teacher Resource pages 28–29* *Student Resource pages 20–25*
If students struggle with Questions 1 to 4	use Pathway 3: Multiplying Whole Numbers *Teacher Resource pages 30–31* *Student Resource pages 26–31*

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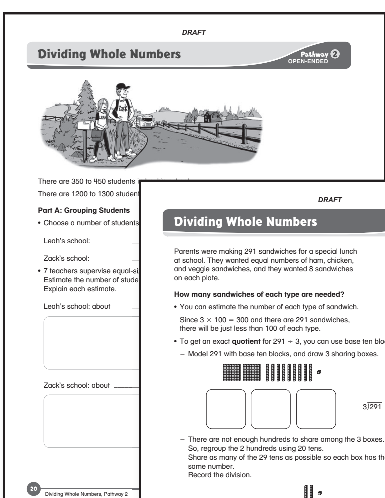
Step 2: Select the intervention pathway

Teacher's Resource



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Dividing Whole Numbers

Pathway 2 OPEN-ENDED

There are 350 to 450 students i…

There are 1200 to 1300 studen…

Part A: Grouping Students

- Choose a number of students…

  Leah's school: ________

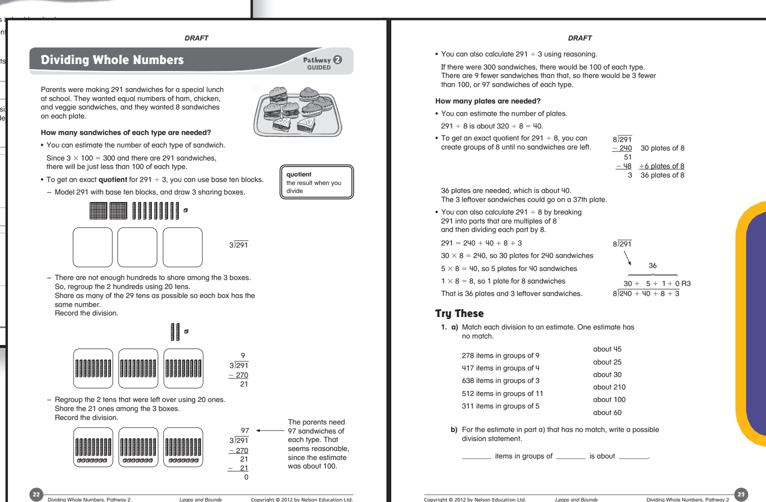
  Zack's school: ________

- 7 teachers supervise equal-si… Estimate the number of stude… Explain each estimate.

  Leah's school: about ________

  Zack's school: about ________

20 Dividing Whole Numbers, Pathway 2



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Dividing Whole Numbers

Pathway 2 GUIDED

Parents were making 291 sandwiches for a special lunch at school. They wanted equal numbers of ham, chicken, and veggie sandwiches, and they wanted 8 sandwiches on each plate.

How many sandwiches of each type are needed?

- You can estimate the number of each type of sandwich.

  Since 3 × 100 = 300 and there are 291 sandwiches, there will be just less than 100 of each type.

- To get an exact **quotient** for 291 ÷ 3, you can use base ten blocks.
  - Model 291 with base ten blocks, and draw 3 sharing boxes.
  - There are not enough hundreds to share among the 3 boxes. So, regroup the 2 hundreds using 20 tens. Share as many of the 29 tens as possible so each box has the same number. Record the division.
  - Regroup the 2 tens that were left over using 20 ones. Share the 21 ones among the 3 boxes. Record the division.

quotient the result when you divide

$$3\overline{)291}$$

$$\begin{array}{r} 9 \\ 3\overline{)291} \\ -\underline{270} \\ 21 \end{array}$$

$$\begin{array}{r} 97 \\ 3\overline{)291} \\ -\underline{270} \\ 21 \\ -\underline{21} \\ 0 \end{array}$$

The parents need 97 sandwiches of each type. That seems reasonable, since the estimate was about 100.

22 Dividing Whole Numbers, Pathway 2   *Leaps and Bounds*   Copyright © 2012 by Nelson Education Ltd.

DRAFT

- You can also calculate 291 ÷ 3 using reasoning.

  If there were 300 sandwiches, there would be 100 of each type. There are 9 fewer sandwiches than that, so there would be 3 fewer than 100, or 97 sandwiches of each type.

How many plates are needed?

- You can estimate the number of plates.

  291 ÷ 8 is about 320 ÷ 8 = 40.

- To get an exact quotient for 291 ÷ 8, you can create groups of 8 until no sandwiches are left.

$$\begin{array}{rl} 8\overline{)291} & \\ -\underline{240} & 30 \text{ plates of } 8 \\ 51 & \\ -\underline{48} & \underline{+6 \text{ plates of } 8} \\ 3 & 36 \text{ plates of } 8 \end{array}$$

  36 plates are needed, which is about 40. The 3 leftover sandwiches could go on a 37th plate.

- You can also calculate 291 ÷ 8 by breaking 291 into parts that are multiples of 8 and then dividing each part by 8.

  291 = 240 + 40 + 8 + 3

  30 × 8 = 240, so 30 plates for 240 sandwiches

  5 × 8 = 40, so 5 plates for 40 sandwiches

  1 × 8 = 8, so 1 plate for 8 sandwiches

  That is 36 plates and 3 leftover sandwiches.

$$8\overline{)291}$$

$$\overbrace{30 + 5 + 1 + 0}^{36}\text{ R3}$$

$$8\overline{)240 + 40 + 8 + 3}$$

Try These

1. a) Match each division to an estimate. One estimate has no match.

278 items in groups of 9	about 45
417 items in groups of 4	about 25
638 items in groups of 3	about 30
512 items in groups of 11	about 210
311 items in groups of 5	about 100
	about 60

   b) For the estimate in part a) that has no match, write a possible division statement.

   ______ items in groups of ______ is about ______.

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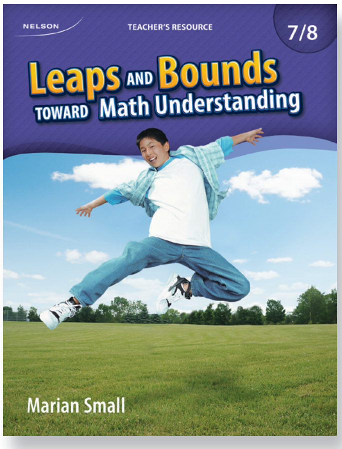
Student Resource

Step 3: Choose an open-ended intervention or guided intervention based on your students' learning preferences or your instructional situation

For more information and full Table of Contents,

visit www.nelson.com/leapsandbounds

Leaps and Bounds is available in a variety of formats to suit your needs!



Teacher's Resource includes:

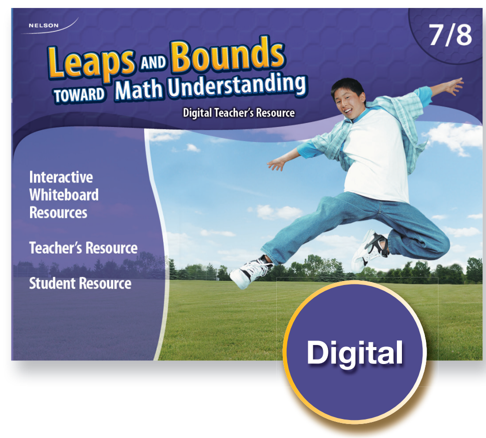
Your choice of print or digital!

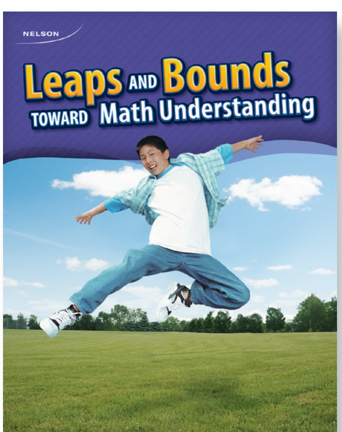
- Diagnostic assessment tools to precisely identify gaps in students' understanding
- Background information on why students might struggle and what misconceptions are revealed by the diagnostic tool
- Up to 5 pathways providing open-ended intervention and more guided intervention for each student or group of students
- Teaching notes to support differentiated instruction, including good questions

Digital Teacher's Resource includes:

Digital format includes Interactive Whiteboard activities!

- Complete Teacher's Resource in PDF format
- SMART™-accredited interactive whiteboard activities and games providing extra practice and additional instructional support
- PowerPoint versions of all interactive whiteboard activities
- Illustrations from Student Resource





Student Resource includes:

Your choice of consumable or Blackline Masters or CD-ROM!

- Intervention activities for topics in every strand to support students who are working as many as 3 levels below grade
- Built-in tips and visuals to support student understanding
- Simple, clear language accessible to ELL students

Dr. Marian Small

Senior Author of *Leaps and Bounds Toward Math Understanding*

"Research has shown that underachieving students improve both in settings that emphasize explicit instruction and modelling, and in settings where students tackle more challenging problems in their own ways. In Leaps and Bounds*, we recognize the value of both approaches—marrying conceptually-clear modelling and practice with open questions that allow students to think more broadly, so that every student has an opportunity to achieve success."*

Contents

Leaps and Bounds 7/8 covers all math strands. This sampler includes Topics 1 and 2 from the Number strand.

Program Overview

Teaching Notes

Strand: Number

Strand: Patterns and Algebra

Strand: Geometry

Strand: Measurement

Strand: Data and Probability

Strand: Number

Number Strand Overview

How were the number topics chosen?

This resource provides materials for assisting students with 10 number topics. These topics were drawn from the curriculum outcomes from across the country for Grades 5 to 7. Topics are divided into distinct levels—called *pathways*—that address gaps in students' prerequisite skills and knowledge.

How were the pathways determined?

At the Grade 7/8 level, the focus moves from whole numbers to decimals, fractions, integers, and proportional reasoning. However, there remains some attention to whole numbers in these materials, specifically, representing and comparing large whole numbers, divisibility, and whole number operations.

There are 3 pathways related to using large numbers. Two involve interpreting numbers in the hundred thousands and numbers greater than 1 million written in standard form; the third helps students see how to use decimals with million and billion benchmarks to describe large whole numbers, for example, 3.2 million.

Work on multiplication of whole numbers goes as far back as multiplying 2 two-digit numbers and multiplying one-digit numbers by multi-digit numbers. A separate pathway focuses on dividing multi-digit numbers by one-digit numbers. Another pathway addresses order of operations with whole numbers.

Another whole numbers topic focuses on factors and multiples of relatively simple whole numbers, prime numbers and perfect squares, and divisibility rules.

Decimals are addressed in 3 topics: representing and comparing decimals, decimal operations, and relating real-life problem situations to the use of decimal operations. The simplest pathway in the topic of representing and comparing decimals involves multiplying decimal tenths and hundredths by powers of 10; the focus is on using place value concepts and explanations. In representing decimals, the materials go back to representing decimal thousandths, but not specifically back to hundredths or tenths. *Leaps and Bounds 5/6* may be used for students needing earlier work. The pathways progress to comparing decimal thousandths and then understanding decimals less than 1 thousandth. Not all pathways are required in all curriculums.

The topic on decimal operations includes adding and subtracting decimals, primarily—but not exclusively—decimal thousandths. There are also pathways involving multiplying decimals by whole numbers, dividing decimals by whole numbers, and dividing whole numbers by simple decimals. Not all pathways are required in all curriculums.

A separate topic helps students recognize the operation(s) suggested by a problem situation. The pathways focus on the different meanings of subtraction (e.g., takeaway or comparison), multiplication (e.g., equal groups or area), and division (e.g., how many groups or how many in each group) in situations involving decimal numbers. There is no separate pathway for addition, since recognizing addition situations is rarely difficult for students.

Two topics focus on fractions. One emphasizes fraction comparisons and the other emphasizes fraction operations. The pathways involving fraction comparisons go back to showing equivalence and then move to the comparison of proper fractions, improper fractions, and mixed numbers. Multiplication and division of fractions are not fully addressed, since they are generally Grade 8 topics, but repeated addition and repeated subtraction of fractions are addressed, as are adding and subtracting fractions and mixed numbers.

The proportional reasoning topic highlights ratios, rates, and percents. There is a pathway for each topic.

The integers topic covers representing and comparing integers and also includes separate pathways for adding integers and subtracting integers.

What number topics are omitted?

Basic multiplication facts and strategies, mental math with whole numbers, and addition and subtraction of whole numbers are dealt with in both *Leaps and Bounds 3/4* and *5/6*, so they were not repeated here as pathways.

Representing and comparing decimal tenths and hundredths is embedded in the pathways dealing with thousandths. If students need even more work with simpler decimals, they could be directed to the *Leaps and Bounds 5/6* material.

Materials

The materials for assisting students struggling with number topics are likely in the classroom or easily accessible. Blackline masters are at the back of this resource.

- base ten blocks
- play coins
- metre sticks
- measuring tapes
- counters
- square tiles
- containers, spoons, and millilitre and litre measures (optional)
- calculators
- pattern blocks
- fraction materials

BLM 1: Millions and Billions
BLM 2: Place Value Charts (to Billions)
BLM 3: Place Value Charts (to Hundred Thousands)
BLM 4: Place Value Charts (to Millionths)
BLM 5: Place Value Charts (to Ten Thousandths)
BLM 6: Place Value Charts (to Thousandths)
BLM 7: Thousandths Grids
BLM 8: Hundredths Grids
BLM 9: Tenths Grids
BLM 10: Fraction Strips
BLM 11: 2 cm Grid Paper
BLM 12: 1 cm Grid Paper
BLM 18: Fraction Circles/Spinners
BLM 19–24: Pattern Blocks

Number Topics and Pathways

Topics and pathways in this strand are shown below.
Each pathway has an open-ended intervention and a guided intervention.

Representing Large Whole Numbers
- Pathway 1: Using Decimals for Large Whole Numbers (TR page 14)
- Pathway 2: Representing Millions and Billions (TR page 16)
- Pathway 3: Representing Six-Digit Numbers (TR page 18)

Whole Number Operations
- Pathway 1: Order of Operations (TR page 26)
- Pathway 2: Dividing Whole Numbers (TR page 28)
- Pathway 3: Multiplying Whole Numbers (TR page 30)

Representing and Comparing Decimals
- Pathway 1: Decimals with Many Places (TR page 40)
- Pathway 2: Comparing Decimal Thousandths (TR page 42)
- Pathway 3: Representing Decimal Thousandths (TR page 44)
- Pathway 4: Multiplying and Dividing by 10, 100, 1000 (TR page 46)

Decimal Operations

- Pathway 1: Dividing Whole Numbers by Decimals (TR page 56)
- Pathway 2: Dividing Decimals by Whole Numbers (TR page 58)
- Pathway 3: Multiplying with Decimals (TR page 60)
- Pathway 4: Adding and Subtracting Decimals (TR page 62)

Relating Situations to Operations

- Pathway 1: Recognizing Division Situations (TR page 70)
- Pathway 2: Recognizing Multiplication Situations (TR page 72)
- Pathway 3: Recognizing Subtraction Situations (TR page 74)

Comparing Fractions

- Pathway 1: Improper Fractions and Mixed Numbers (TR page 82)
- Pathway 2: Proper Fractions (TR page 84)
- Pathway 3: Equivalent Fractions (TR page 86)

Fraction Operations

- Pathway 1: Repeated Addition of Fractions (TR page 98)
- Pathway 2: Adding and Subtracting Mixed Numbers (TR page 100)
- Pathway 3: Subtracting Fractions (TR page 102)
- Pathway 4: Adding Fractions (TR page 104)

Rates, Percents, and Ratios

- Pathway 1: Using Rates (TR page 112)
- Pathway 2: Using Percents (TR page 114)
- Pathway 3: Using Ratios (TR page 116)

Multiplicative Relationships

- Pathway 1: Divisibility Rules (TR page 124)
- Pathway 2: Prime Numbers and Perfect Squares (TR page 126)
- Pathway 3: Factors and Multiples (TR page 128)

Integers

- Pathway 1: Subtracting Integers (TR page 136)
- Pathway 2: Adding Integers (TR page 138)
- Pathway 3: Representing and Comparing Integers (TR page 140)

Strand: Number

Representing Large Whole Numbers

Planning For This Topic

Materials for assisting students with representing, comparing, and renaming large numbers consist of a diagnostic tool and 3 intervention pathways. Pathway 1 focuses on the use of decimals to represent large whole numbers (e.g., 3 300 000 as 3.3 million). Pathway 2 focuses on standard form for numbers greater than 1 million. Pathway 3 focuses on standard form for six-digit numbers.

Each pathway has an open-ended intervention and a guided intervention. Choose the type of intervention more suitable for your students' needs and your particular circumstances.

Curriculum Connections

Grades 5 to 8 curriculum connections for this topic are provided online. See www.nelson.com/leapsandbounds. The Ontario curriculum does not specifically mention place value for numbers beyond 1 million, so Pathways 1 and 2 may be viewed as optional for these students. The WNCP curriculum extends whole number representations to 1 million in Grade 5 and greater than 1 million in Grade 6. Neither curriculum specifically states that students need to interpret and use large whole numbers written using decimals (Pathway 1). However, these activities are helpful for students in everyday life and in work they do with such numbers in science.

Professional Learning Connections

PRIME: Number and Operations, Background and Strategies (Nelson Education, 2005), pages 63–68

Making Math Meaningful to Canadian Students K–8 (Nelson Education Ltd., 2008), pages 137–144, 146–147

Big Ideas from Dr. Small, Grades 4–8 (Nelson Education Ltd., 2010), pages 15–19

More Good Questions (dist. by Nelson Education Ltd., 2010), page 67

Why might students struggle with large numbers?

Students might struggle with large numbers for any of the following reasons:

- Students might have little, if any, concrete experience and limited life experience with large numbers, whether in standard form or in decimal form (e.g., 1.1 million).
- Students might not know where certain large whole numbers might appear in context (e.g., the number of people in India is in the billions place).
- Written conventions for numbers are based on place value and not on the way the number word sounds (e.g., "one million three hundred" is not 1 000 000 300).
- Students might not understand the periodic nature of the place value system, so it is not obvious why you read 123 400 as *123 thousand 400*.
- They might not know where to put the spaces in large whole numbers; that is, they do not understand the role of the periods in reading and writing numerals (e.g., they might write *forty-seven thousand twelve* as 47 000 12).
- It is not intuitively obvious why the value of a digit changes depending on its place in a numeral (e.g., why the value of the 3 in 302 415 is different from the value of 3 in 203 415).

- Some students struggle with representations of whole numbers other than standard form (e.g., they might find it difficult to write or think of 3 200 000 as 3200 thousands).
- Students might be unfamiliar with the use of large number units, such as millions or billions, in combination with decimal values.
- Students who are not generally comfortable with decimals may be reluctant to use decimals to represent whole numbers.

Diagnostic Tool: Representing Large Whole Numbers

Use the diagnostic tool to determine the most suitable intervention pathway for representing large whole numbers. Provide Diagnostic Tool: Representing Large Whole Numbers, Teacher's Resource pages 8 to 10, and have students complete it in writing or orally. If students need more place value charts, provide Place Value Charts (to Billions) (BLM 2).

See solutions on Teacher's Resource pages 11 to 13.

Intervention Pathways

These intervention pathways help students work with various ways of representing large whole numbers. This prepares them for working flexibly with large whole numbers of any size.

There are 3 pathways:

- Pathway 1: Using Decimals for Large Whole Numbers
- Pathway 2: Representing Millions and Billions
- Pathway 3: Representing Six-Digit Numbers

Use the chart below (or the Key to Pathways on Teacher's Resource pages 11 to 13) to determine which pathway is most suitable for each student or group of students.

Diagnostic Tool Results	Intervention Pathway
If students struggle with Questions 1e–f, 2e–f, 3c, 4e–f, 5e–f, 6e–f, 7e–f, 8c	use Pathway 1: Using Decimals for Large Whole Numbers *Teacher Resource pages 14–15* *Student Resource pages 1–5*
If students struggle with Questions 1c–d, 2c–d, 3b, 4c–d, 5c–d, 6c–d, 7c–d, 8b	use Pathway 2: Representing Millions and Billions *Teacher Resource pages 16–17* *Student Resource pages 6–11*
If students struggle with Questions 1a–b, 2a–b, 3a, 4a–b, 5a–b, 6a–b, 7a–b, 8a	use Pathway 3: Representing Six-Digit Numbers *Teacher Resource pages 18–19* *Student Resource pages 12–15*

Name: ____________________ DRAFT ______ Date: ____________________

Representing Large Whole Numbers

Diagnostic Tool

You can use a place value chart like this to help you answer Questions 1 to 6.

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O

1. What do the digits 4 and 7 represent in each number?

a) **427** 213 ____________ ____________

b) 1**4**3 **7**28 ____________ ____________

c) **4** 2**7**3 689 ____________ ____________

d) 2**4**1 1**7**6 538 916 ____________ ____________

e) **4.7** million ____________ ____________

f) 2.**47** billion ____________ ____________

2. Write each number in standard form.

a) 4 hundred thousand + 24 ____________

b) 215 thousand + 46 ____________

c) 3 million + 40 thousand + 100 ____________

d) 12 billion + 17 thousand + 10 ____________

e) 4.2 million ____________

f) 3.71 billion ____________

3. How are the values of the two 6s in each number different?

a) **6**00 0**6**0 ________________________

b) **6**0 00**6** 030 ________________________

c) **6**.2**6** million ________________________

Name:________________________DRAFT________ Date:________________________

4. Create 2 numbers to match each description.

a) 6 in the hundred thousands place and 2 in the hundreds place

_______________ _______________

b) 6 in the ten thousands place and 2 in the tens place

_______________ _______________

c) 4 in the ten millions place and 5 in the thousands place

_______________ _______________

d) 1 in the billions place and 2 in the hundred millions place

_______________ _______________

e) about 1 hundred thousand more than 2.4 million

_______________ _______________

f) about 100 million less than 4.62 billion

_______________ _______________

5. Fill in the blank with a number or a place value word.

a) 3 hundred thousand = ________ thousand

b) 40 ten thousand = 4 ________________

c) 2 million = ________ thousand

d) 30 billion = 3000 ________________

e) 4.3 billion = ________ million

f) 2.1 million = 2100 ________________

6. Estimate.

a) 461 589 is about ________ hundred thousand.

b) 304 127 is about ________ thousand.

c) 12 089 157 is about ________ million.

d) 11 314 578 138 is about ________ million.

e) 0.7 million is about ________ hundred thousand.

f) 1.77 billion is about ________ million.

Name: ____________________ **DRAFT** ______ Date: ____________________

You can use this place value chart to help you answer Question 7.

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O

7. Order each set of numbers from least to greatest.

a) 304 209   940 302   94 302

_______________ , _______________ , _______________

b) 38 158   624 135   622 143

_______________ , _______________ , _______________

c) 3 121 043   31 043 699   3 112 043

_______________ , _______________ , _______________

d) 4 002 003 700   384 674 121   583 912 934

_______________ , _______________ , _______________

e) 3.4 million   1.3 billion   9.9 million

_______________ , _______________ , _______________

f) 8.03 million   0.83 billion   0.5 million

_______________ , _______________ , _______________

8. Match each number to what you think it most likely describes.

312 678	population of China
1 300 000 000	population of Canada
33.3 million	cost of a house in dollars

Solutions and Key to Pathways

Name:______________________ Date:______________

Representing Large Whole Numbers

Diagnostic Tool

You can use a place value chart like this to help you answer Questions 1 to 6.

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O

1. What do the digits 4 and 7 represent in each number?

Pathway 3

a) **4**2**7** 213 — 400 000 — 7000

b) 1**4**3 **7**28 — 40 000 — 700

Pathway 2

c) **4** 2**7**3 689 — 4 000 000 — 70 000

d) 2**4**1 1**7**6 538 916 — 40 000 000 000 — 70 000 000

Pathway 1

e) **4**.**7** million — 4 000 000 — 700 000

f) 2.**47** billion — 400 000 000 — 70 000 000

Note: Answers using place value language are also correct, e.g. 4 hundred thousand and 7 thousand for Question 1a).

2. Write each number in standard form.

Pathway 3

a) 4 hundred thousand + 24 — 400 024

b) 215 thousand + 46 — 215 046

Pathway 2

c) 3 million + 40 thousand + 100 — 3 040 100

d) 12 billion + 17 thousand + 10 — 12 000 017 010

Pathway 1

e) 4.2 million — 4 200 000

f) 3.71 billion — 3 710 000 000

3. How are the values of the two 6s in each number different?

Pathway 3

a) **6**00 0**6**0 — 6 hundred thousands versus 6 tens

Pathway 2

b) **6**0 00**6** 030 — 6 ten (or 60) millions versus 6 thousands

Pathway 1

c) **6**.2**6** million — 6 millions versus 6 ten (or 60) thousands

8 Representing Large Whole Numbers, Diagnostic Tool *Leaps and Bounds*

Name:________________________________ Date:__________________

4. Create 2 numbers to match each description.

Pathway 3

a) 6 in the hundred thousands place and 2 in the hundreds place

615 223 600 200

b) 6 in the ten thousands place and 2 in the tens place

162 024 64 028

Pathway 2

c) 4 in the ten millions place and 5 in the thousands place

43 005 234 47 225 111

d) 1 in the billions place and 2 in the hundred millions place

1 200 000 000 1 223 456 789

Pathway 1

e) about 1 hundred thousand more than 2.4 million

2 500 000 2 500 010

f) about 100 million less than 4.62 billion

4 520 220 120 4 520 110 345

5. Fill in the blank with a number or a place value word.

Pathway 3

a) 3 hundred thousand = 300 thousand

b) 40 ten thousand = 4 hundred thousand

Pathway 2

c) 2 million = 2000 thousand

d) 30 billion = 3000 ten million

Pathway 1

e) 4.3 billion = 4300 million

f) 2.1 million = 2100 thousand

6. Estimate.

Pathway 3

a) 461 589 is about 5 hundred thousand.

b) 304 127 is about 300 thousand.

Pathway 2

c) 12 089 157 is about 12 million.

d) 11 314 578 138 is about 11 300 million.

Pathway 1

e) 0.7 million is about 7 hundred thousand.

f) 1.77 billion is about 1800 million.

Name:_______________________________ Date:_______________

You can use this place value chart to help you answer Question 7.

Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O

7. Order each set of numbers from least to greatest.

Pathway 3

a) 304 209   940 302   94 302

94 302, 304 209, 940 302

b) 38 158   624 135   622 143

38 158, 622 143, 624 135

Pathway 2

c) 3 121 043   31 043 699   3 112 043

3 112 043, 3 121 043, 31 043 699

d) 4 002 003 700   384 674 121   583 912 934

384 674 121, 583 912 934, 4 002 003 700

Pathway 1

e) 3.4 million   1.3 billion   9.9 million

3.4 million, 9.9 million, 1.3 billion

f) 8.03 million   0.83 billion   0.5 million

0.5 million, 8.03 million, 0.83 billion

8. Match each number to what you think it most likely describes.

Pathway 3
Pathway 2
Pathway 1

312 678	population of China
1 300 000 000	population of Canada
33.3 million	cost of a house in dollars

Pathway 1
OPEN-ENDED

Using Decimals for Large Whole Numbers

You will need

- base ten blocks (small cube, rod, flat, large cube)
- Place Value Charts (to Billions) (BLM 2)
- Internet access, or Millions and Billions (BLM 1)
- Student Resource pages 1–2

Open-Ended Intervention

Before Using the Open-Ended Intervention

Write the decimal 0.1 and have students read it aloud (*one tenth*). Show a base ten small cube, rod, flat, and large cube and ask:

- Suppose the rod was the whole. What would represent 0.1? Why? (*a small cube; it is one tenth of 10*)
- Suppose the flat was the whole? What would represent 0.1? Why? (*a rod; 10 rods make 1 flat*)
- Why did the decimal value change? (*The value of the whole changed.*)

Draw a place value chart to thousands and have students record 3200 in it. Ask:

- What is the value of the digit 3? the digit 2? (*3 thousands; 2 hundreds*)
- Why might someone write 3200 as 3.2 thousand? (e.g., *If the large cube is the whole, 100 is 0.1 of a thousand, so 200 is 0.2, so the 200 in 3200 is 0.2 and the 3000 is 3.*)

Using the Open-Ended Intervention **Student Resource pages 1–2**

Read through the tasks on the student pages together. Provide place value charts and allow for Internet access (using search words such as "populations," "popular products," "sales," "space distances," "sports salaries," and "downloads") or provide Millions and Billions (BLM 1). Give students time to work, ideally in pairs.

Observe whether students

- can use standard form to rename a large whole number expressed in decimal form and vice versa
- can order large whole numbers expressed in decimal form
- understand that a number such as 12.3 million (a large whole number expressed in decimal form) is a concise way to write a large whole number
- recognize real-life contexts for large whole numbers expressed in decimal form

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- How did you decide what a decimal portion of a million represented? (e.g., *If it was 1 place to the right of the millions, I knew it was tenths of it, but 2 places to the right meant it was hundredths of it.*)
- Why did the numbers in standard form have a lot of zeros? (e.g., *A number like 2.3 million has non-zero digits only in the millions and hundred thousands places. It's like writing 2.300 000 million.*)
- How did you order the 10 numbers? (e.g., *First, I ordered the millions by putting them in decimal form, then I ordered the billions in decimal form, and I then realized all the million numbers were less than all the billion numbers.*)

Using Decimals for Large Whole Numbers

Pathway 1 GUIDED

Guided Intervention

You will need

- base ten blocks (small cube, rod, flat, large cube)
- Place Value Charts (to Billions) (BLM 2)
- Internet access, or Millions and Billions (BLM 1)
- Student Resource pages 3–5

Before Using the Guided Intervention

Write the decimal 0.1 and have students read it aloud (*one tenth*). Show a base ten small cube, rod, flat, and large cube and ask:

- Suppose the rod was the whole. What would represent 0.1? Why? (*a small cube; it is 1 tenth of 10*)
- Suppose the flat was the whole? What would represent 0.1? Why? (*a rod; 10 rods make 1 flat*)
- Why did the decimal value change? (*The value of the whole changed.*)

Draw a place value chart to thousands and have students record 3200 in it. Ask:

- What is the value of the digit 3? the digit 2? (*3 thousands; 2 hundreds*)
- Why might someone write 3200 as 3.2 thousand? (e.g., *If the large cube is the whole, 100 is 0.1 of a thousand, so 200 is 0.2, so the 200 in 3200 is 0.2 and the 3000 is 3.*)

Using the Guided Intervention **Student Resource pages 3–5**

Provide place value charts and work through the instructional section of the student pages together.

Have students work through the **Try These** questions in pairs or individually. They will need Internet access or Millions and Billions (BLM 1), for Question 7.

Observe whether students

- can change from decimal form to standard form and vice versa (Questions 1, 2, 3)
- can compare and order large whole numbers in decimal form (Question 4)
- recognize the value of a specific digit (Question 5)
- can rename a large whole number in other forms (Question 6)
- recognize contexts for large whole numbers (Question 7)
- recognize the value of writing large whole numbers in decimal form (Question 8)

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- Why did the numbers in Question 1 have a lot of zeros? (e.g., *A number like 4.13 billion has non-zero digits only in the billions, hundred millions, and ten millions places. It's like writing 4.130 000 000 billion.*)
- How did you determine the decimal forms in Question 3? (e.g., *For 12 345 000, I wrote the number of millions, 12, as the whole, and then the other part, 345, as the decimal.*)
- Why do people write large whole numbers as decimal millions or billions? (e.g., *It takes up less space and you're usually interested only in the digits that have big place values anyway, because they tell you how big the number is.*)

Pathway 2
OPEN-ENDED

Representing Millions and Billions

You will need

- Place Value Charts (to Billions) (BLM 2)
- Student Resource pages 6–7

Open-Ended Intervention

Before Using the Open-Ended Intervention

Write the numeral 739 907 and read it aloud (*739 thousand 907*). Sketch a place value chart to hundred thousands and label the thousands period. Write 739 907 in the chart and read it aloud. Ask:

- ▸ How does the chart help you read the number? (e.g., *The chart shows the number in 2 parts: a number of thousands and a number less than a thousand. Each part has 3 place value columns.*)
- ▸ How do you know 740 907 is more than 739 907 but 739 897 is less than 739 907? (e.g., *740 907 has more ten thousands than 739 897, and 739 897 has fewer hundreds than 739 907.*)
- ▸ How would you show 999 thousand 999? (*9 in each of the 6 columns*)
- ▸ What would happen if you added 1 to 999 999? (e.g., *I'd have 1000 thousand, and I would need a new column on the left.*)

Using the Open-Ended Intervention Student Resource pages 6–7

Read through the tasks on the student pages together. Ensure that students understand the periods shown on the place value chart. Provide place value charts and give students time to work, ideally in pairs.

Observe whether students

- can create numbers that are about 111 million and write them in standard form
- can create numbers very close to 278 billion and write them in standard form
- can order large numbers and explain their thinking
- can read, estimate, and relate large numbers

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- ▸ How did you decide that a number was close to 111 million? (e.g., *It was close but greater if it was 111 million and not too many thousands more. It was close but less if it was 110 million and more than 900 thousand more.*)
- ▸ How did you decide if a number was close to 278 billion? (e.g., *It was close if it was less than 1 million more than 278 billion or if it was a lot of hundreds of millions more than 277 billion.*)
- ▸ How did you arrange the numbers in order? (e.g., *I started with the numbers in millions—110 million is less than 111 million. If the millions parts were the same, then I compared the thousands. Then I used the billions numbers and compared their millions parts, and sometimes their thousands or even their ones parts.*)
- ▸ In order, all the million values come first and then all the billion values. Why does that make sense? (e.g., *278 billion is much more than 111 million, so anything close to it has to be more than something close to 111 million.*)

Representing Millions and Billions

Pathway 2
GUIDED

Guided Intervention

Before Using the Guided Intervention

Write the numeral 739 907 and read it aloud (*739 thousand 907*). Sketch a place value chart to hundred thousands and label the thousands period. Ask:

- Write 739 907 in the chart and read it aloud. How does the chart help you read the number?
  (e.g., *The chart shows that the number is in 2 parts: a number of thousands and a number less than a thousand. Each part has 3 place value columns.*)

Using the Guided Intervention **Student Resource pages 8–11**

Provide place value charts and work through the instructional section of the student pages together. Ensure that students are comfortable using the place value chart and reading and representing numbers to billions. Students will need Internet access or Millions and Billions (BLM 1) for Question 11.

Have students work through the **Try These** questions in pairs or individually.

Observe whether students
- can read large numbers (Questions 1, 6)
- can rename large numbers (Questions 2, 9)
- can compare large numbers and explain their thinking (Questions 3, 8, 10)
- can estimate large numbers (Question 4)
- can relate 2 large numbers (Questions 5, 12)
- recognize the value of a digit in a large number (Question 7)
- recognize real-life contexts for large numbers (Question 11)

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:
- How did you know 153 million + 32 thousand was less than 1 billion in Question 3?
  (e.g., *A billion is 1000 million, and 153 million + 32 thousand is not even 154 million.*)
- How did you decide that 4 128 756 was about 4100 thousand in Question 4?
  (e.g., *4 million = 4000 thousand, so I added that to 100 000 in the next place to the right.*)
- Which digit changed in Question 5d)? Why?
  (*8*; e.g., *If you add 2 million to 8 million, you get 10 million, so the million and ten million digits changed.*)
- Why was the number of hundred thousands less than the number of thousands in Question 9a)?
  (e.g., *Hundred thousands is a bigger place value unit than thousands, and if the unit is bigger, fewer units are needed to represent the same number.*)

You will need

- Place Value Charts (to Billions) (BLM 2)
- Millions and Billions (BLM 1)
- Student Resource pages 8–11

Pathway 3 OPEN-ENDED

Representing Six-Digit Numbers

You will need

- Place Value Charts (to Hundred Thousands) (BLM 3)
- Student Resource page 12

Open-Ended Intervention

Before Using the Open-Ended Intervention

Sketch a place value chart to hundred thousands. Ask:

- How would you show 20 000 on the chart?
  (*2 in the ten thousands column and then all zeros*)
- Write 3 numbers that are close to 20 000. Make 1 number less than 20 000.
  (e.g., *20 001, 20 100, 19 999*)
- Order the 3 numbers and explain how you did it.
  (e.g., *19 999, 20 001, 20 100; 19 999 is 1 less than 20 000, so it's lowest; 20 001 is 1 more and 20 100 is 100 more.*)
- How would you show 90 000 on the chart?
  (*9 in the ten thousands column and then all zeros*)
- What would happen if you added 10 000 to 90 000?
  (e.g., *There would be 10 in the ten thousands column, so I would trade it for 1 hundred thousand.*)

Using the Open-Ended Intervention **Student Resource page 12**

Read through the tasks on the student page together. Provide place value charts. Give students time to work, ideally in pairs.

Observe whether students

- can create large numbers of a given size and write them in standard form
- can estimate large numbers
- can order large numbers in standard form and explain their thinking

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- How did you decide that a number was close to 750 thousand?
  (e.g., *It was close but greater if it was less than 1 thousand more than 750 thousand, or it was close but less if it was a lot of hundreds more than 749 thousand.*)
- What did you do to make the number close to but less than 750 thousand? Why?
  (e.g., *I used 749 thousand instead of 750 thousand, plus a lot of hundreds and some tens and ones.*)
- How did you arrange the numbers in order?
  (e.g., *I started with the number of thousands—749 is less than 750. Then within each group, I looked at the rest of the number to see which had a greater group of hundreds, tens, and ones.*)

Representing Six-Digit Numbers

Pathway 3 GUIDED

Guided Intervention

You will need

- Place Value Charts (to Hundred Thousands) (BLM 3)
- Student Resource pages 13–15

Before Using the Guided Intervention

Sketch a place value chart to hundred thousands. Ask:

- How would you show 20 000 on the chart? (*2 in the ten thousands column and then all zeros*)
- Write 3 numbers that are close to 20 000. Make 1 number less than 20 000. (e.g., *20 001, 20 100, 19 999*)
- Order the 3 numbers. How did you do it? (e.g., *19 999, 20 001, 20 100; 19 999 is 1 less than 20 000, so it's lowest; 20 001 is 1 more and 20 100 is 100 more.*)
- How would you show 90 000 on the chart? (*9 in the ten thousands column and then all zeros*)
- What would happen if you added 10 000 to 90 000? (e.g., *There would be a 10 in the ten thousands column, so I would trade it for 1 hundred thousand.*)

Using the Guided Intervention **Student Resource pages 13–15**

Provide place value charts and work through the instructional section on the student page together.

Have students work through the **Try These** questions in pairs or individually.

Observe whether students

- can read large numbers (Questions 1, 8)
- can write a number in standard form (Question 2)
- can compare and relate numbers (Questions 3, 5, 7)
- can estimate large numbers (Question 4)
- recognize the value of a digit in a large number (Question 6)
- can rename large whole numbers (Question 9)
- recognize real-life contexts for large numbers (Question 10)

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- How did you know that 80 thousand was less than 20 ten thousand in Question 3? (e.g., *80 thousand is 8 ten thousand.*)
- How did you decide that 493 127 was about 5 hundred thousand in Question 4? (e.g., *I knew that 493 thousand was closer to 500 thousand than to 400 thousand and that 500 thousand is 5 hundred thousand.*)
- Which digits changed in Question 5c)? Why? (*8 and 3*; e.g., *If you add 8 thousand to 3 thousand, you get 11 thousand, or 1 ten thousand and 1 thousand, so 3 thousand changed to 1 thousand and 8 ten thousand plus the 1 ten thousand is 9 ten thousand.*)
- How did you pick the greater number in Question 7d)? (e.g., *711 234 is a lot of hundred thousands and 79 929 is not even 1 hundred thousand.*)

Whole Number Operations

Planning For This Topic

Materials for assisting students with whole number operations, particularly multiplication and division, consist of a diagnostic tool and 3 intervention pathways. Pathway 1 involves the use of order of operations with whole numbers. Pathway 2 focuses on division of three-digit and four-digit numbers by one-digit and two-digit numbers. Pathway 3 involves multiplication of two-digit by two-digit numbers.

Each pathway has an open-ended intervention and a guided intervention. Choose the type of intervention more suitable for your students' needs and your particular circumstances.

Curriculum Connections

Grades 5 to 8 curriculum connections for this topic are provided online. See www.nelson.com/leapsandbounds. Both the Ontario and WNCP curricula include multiplying and dividing with whole numbers in Grades 5 and 6. The strategies mentioned include using technology, mental math strategies, and paper and pencil or pictorial/concrete models. The technology is not specifically addressed here, but the conceptual understanding involved in mental math and other strategies is. Both curricula refer to the need for students to understand and apply the order of operations rules with whole numbers. Although calculator use would be expected with complicated numbers, the values used here are simple enough that calculators are not required. There is no specific mention of multiplication and division of whole numbers in Grade 7 or 8; however, students continue to use these operations to solve problems.

Professional Learning Connections

PRIME: Number and Operations, Background and Strategies (Nelson Education Ltd., 2005), pages 82–97
Making Math Meaningful to Canadian Students K–8 (Nelson Education Ltd., 2008), pages 173–189
Big Ideas from Dr. Small Grades 4–8 (Nelson Education Ltd., 2010), pages 25–41
Good Questions (dist. by Nelson Education Ltd., 2009), pages 27, 29

Why might students struggle with operations?

Students might struggle with multiplying and dividing whole numbers and applying the order of operations for any of the following reasons:

- They might not know all of their multiplication facts, making it difficult to perform more complex multiplication or division calculations and making it difficult to estimate or use mental strategies.
- They might multiply 2 two-digit numbers "in columns" like addition (e.g., they might multiply 34×25 by multiplying 3×2 and then appending 4×5 to get 620 instead of 850).
- They might recall procedures learned incorrectly (e.g., multiplying 24×63 by multiplying 4×63 and then putting a 2 in front).
- They might not relate the operations to their various meanings (e.g., division as a sharing or an equal group situation, and multiplication as the area of a rectangle).

- They might not recognize division as the inverse operation of multiplication (e.g., that solving $455 \div 5 = ■$ is the same as solving $5 \times ■ = 455$).
- They might not recognize how to separate the dividend into convenient parts for dividing (e.g., they may not realize that to divide 120 by 9, it might be convenient to think of $90 \div 9 + 30 \div 9$).
- They might not know what to do with remainders in particular contexts.
- They might not realize that different values can result when a sequence of calculations is performed in a different order and that there are accepted conventions for the correct order.

Diagnostic Tool: Whole Number Operations

Use the diagnostic tool to determine the most suitable intervention pathway for whole number operations. Provide Diagnostic Tool: Whole Number Operations, Teacher's Resource pages 22 and 23, and have students complete it in writing or orally. You might provide base ten materials.

Intervention Pathways

The purpose of the intervention pathways is to help students multiply and divide whole numbers and perform operations correctly in a sequence of calculations. The focus is to prepare them for solving problems and working with other types of numbers, such as integers and fractions.

There are 3 pathways:
- Pathway 1: Order of Operations
- Pathway 2: Dividing Whole Numbers
- Pathway 3: Multiplying Whole Numbers

Use the chart below (or the Keys to Pathways on Teacher's Resource pages 24 and 25) to determine which pathway is most suitable for each student or group of students.

Diagnostic Tool Results	Intervention Pathway
If students struggle with Questions 9 to 11	use Pathway 1: Order of Operations *Teacher Resource pages 26–27* *Student Resource pages 16–19*
If students struggle with Questions 5 to 8	use Pathway 2: Dividing Whole Numbers *Teacher Resource pages 28–29* *Student Resource pages 20–25*
If students struggle with Questions 1 to 4	use Pathway 3: Multiplying Whole Numbers *Teacher Resource pages 30–31* *Student Resource pages 26–31*

Name: DRAFT Date:

Whole Number Operations

Diagnostic Tool

1. Circle the calculation with an answer closest to 400.

21 × 24   12 × 34   50 × 81   17 × 25

2. Multiply. Show your work.

a) 32 × 21 = ____________

c) 17 × 73 = ____________

b) 43 × 46 = ____________

d) 15 × 15 = ____________

3. Create a real-life problem that could be solved using 12 × 38.

____

____

4. Show or describe 2 ways to calculate 35 × 17.

5. Circle the calculation with an answer closest to 120.

847 ÷ 7   810 ÷ 6   1368 ÷ 9   56 ÷ 4

6. Calculate. Show your work.

a) $512 \div 6 =$ ____________

c) $812 \div 9 =$ ____________

b) $3009 \div 4 =$ ____________

d) $4220 \div 4 =$ ____________

7. Create a real-life problem that could be solved using $365 \div 7$.

____

____

8. Show or describe 2 ways to calculate $390 \div 6$.

9. In each expression, circle the sign for the calculation you would do first.

a) $4 + 6 \times 5$

c) $3 + 12 \div 3$

b) $6 \times (4 + 5)$

d) $6 + 3 - 4 \times 2$

10. Is $(3 + 5) \times 2$ equal to $3 + 5 \times 2$? How do you know?

____

____

11. Use $>$, $<$, or $=$ to make each statement true.

a) $4 \times 5 \times 6$ ☐ $4 \times (5 \times 6)$

c) $4 + 5 \times 6$ ☐ $(4 + 5) \times 6$

b) $4 \times 5 + 6$ ☐ $4 \times (5 + 6)$

d) $4 \times (3 + 2) \times 5$ ☐ $(4 \times 3) + (2 \times 5)$

Solutions and Key to Pathways

Name:____________________________________ Date:____________________

Whole Number Operations

Diagnostic Tool

Pathway 3

1. Circle the calculation with an answer closest to 400.

21×24 12×34 50×81 17×25

2. Multiply. Show your work.

a) $32 \times 21 =$ 672

e.g., 32×21
$= 32 \times 20 + 32 \times 1$
$= 640 + 32$
$= 672$

c) $17 \times 73 =$ 1241

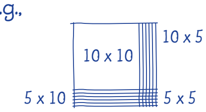
e.g., $17 \times 73 = 20 \times 73 - 3 \times 73$
$= 10 \times 2 \times 73 - 219$
$= 10 \times 146 - 219$
$= 1460 - 219$
$= 1241$

b) $43 \times 46 =$ 1978

e.g.,
$$\begin{array}{r} 43 \\ \times\ 46 \\ \hline 240 \\ 18 \\ 1600 \\ 120 \\ \hline 1978 \end{array}$$

d) $15 \times 15 =$ 225

e.g.,



3. Create a real-life problem that could be solved using 12×38.

e.g., You sell 12 magazine subscriptions and each costs $38.

How much money did you bring in?

4. Show or describe 2 ways to calculate 35×17.

e.g., Multiply 10×17, then triple it. Then take half of 10×17 and add the 2 numbers together.

e.g., Make a rectangle that is 35 units long and 17 units wide and fill it with base ten blocks.

Pathway 2

5. Circle the calculation with an answer closest to 120.

$847 \div 7$ $810 \div 6$ $1368 \div 9$ $56 \div 4$

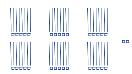
Name:______________________________ Date:________________

Pathway 2

6. Calculate. Show your work.

a) $512 \div 6 =$ 85 R2 or 85.33

e.g.,



c) $812 \div 9 =$ 90 R2 or 90.22

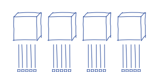
e.g., $812 \div 9$
$= 810 \div 9 + 2 \div 9$
$= 90$ R2

b) $3009 \div 4 =$ 752 R1 or 752.25

e.g., $4 \times 750 = 3000$
$4 \times 2 = 9$
So, 3009 4 = 752 R1

d) $4220 \div 4 =$ 1055

e.g.,



7. Create a real-life problem that could be solved using $365 \div 7$.

e.g., How many weeks are there in 365 days?

8. Show or describe 2 ways to calculate $390 \div 6$.

one way e.g., I divided 390 by 3 in my head. It's 130. Then I divided that in half. 130 is 120 + 10, so half is 60 + 5. The answer is 65.

another way e.g., Write 390 as 360 + 30 and divide each part by 6 and then add the answers. It would be 60 + 5 = 65.

Pathway 1

9. In each expression, circle the sign for the calculation you would do first.

a) $4 + 6 \times 5$

b) $6 \times (4 + 5)$

c) $3 + 12 \div 3$

d) $6 + 3 - 4 \times 2$

10. Is $(3 + 5) \times 2$ equal to $3 + 5 \times 2$? How do you know?

No. e.g., In $(3 + 5) \times 2$, you are doubling both 3 and 5 but in $3 + 5 \times 2$ you are only doubling 5 and then adding it to 3.

11. Use $>$, $<$, or $=$ to make each statement true.

a) $4 \times 5 \times 6 \; \boxed{=} \; 4 \times (5 \times 6)$

b) $4 \times 5 + 6 \; \boxed{<} \; 4 \times (5 + 6)$

c) $4 + 5 \times 6 \; \boxed{<} \; (4 + 5) \times 6$

d) $4 \times (3 + 2) \times 5 \; \boxed{>} \; (4 \times 3) + (2 \times 5)$

 Leaps and Bounds Whole Number Operations, Diagnostic Tool 23

Pathway 1
OPEN-ENDED

Order of Operations

You will need

- Student Resource page 16

Open-Ended Intervention

Before Using the Open-Ended Intervention

Record the numbers 9, 2, and 4. Tell students to subtract 2 from 9 and then multiply by 4. Ask:

- ▸ What is the result? (*28*)

Introduce the term *expression* as a series of calculations involving numbers. Ask:

- ▸ Why can you write what you did in an expression as $(9 - 2) \times 4$, or as $4 \times (9 - 2)$?
  (e.g., *The brackets tell you to calculate $9 - 2$ first so you'd get the same answer for both, since $4 \times 7 = 7 \times 4$.*)
- ▸ Why is $9 - 2 \times 4$ equal to 1 and not 28?
  (e.g., *Since there are no brackets, you multiply 2×4 first and then subtract from 9.*)
- ▸ Describe how to calculate $4 + 3 \times (8 - 6)$.
  (e.g., *Subtract 6 from 8 to get 2, then multiply the 2 by 3 to get 6, and then add the 6 to 4 to get 10.*)

Using the Open-Ended Intervention Student Resource page 16

There is no mention of the E (for exponents) in BEDMAS in the Student Resource. However, you might let students know that later they will use special symbols called exponents, involving repeated multiplication.

Read through the tasks on the student page together. Note that simple numbers are used so that students do not need calculators (since some calculators automatically follow the order of operations for the student). Have students who need calculators use them for individual calculations, not the entire expression. Give students time to work, ideally in pairs.

Observe whether students

- can describe how to perform a sequence of calculations
- can insert brackets into an expression to achieve a given result
- recognize that sometimes the use of brackets is optional
- can communicate clearly how to perform a sequence of operations

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- ▸ Did you write any expressions where the order from left to right would have been the correct order for calculation?
  (*yes*, e.g., *for $(4 \times 5) \div 2 + 8 - 3$*)
- ▸ Why did your strategy for moving brackets to get a greater answer make sense?
  (e.g., *By taking the 36 out of the brackets, it meant that I divided by a smaller number, and I knew that would result in a bigger answer.*)
- ▸ Why are order of operations rules needed?
  (e.g., *Different people might get different answers for an expression involving a lot of operations when that expression was written down and not said.*)

Order of Operations

Pathway 1 GUIDED

Guided Intervention

Before Using the Guided Intervention

Record the numbers 9, 2, and 4. Tell students to subtract 2 from 9 and then multiply by 4. Ask:

- What is the result? (*28*)

Introduce the term *expression* as a series of calculations involving numbers. Ask:

- Why can you write what you did in an expression as $(9 - 2) \times 4$, or as $4 \times (9 - 2)$? (e.g., *The brackets tell you to calculate 9 − 2 first so you'd get the same answer for both, since 4 × 7 = 7 × 4.*)
- Why is $9 - 2 \times 4$ equal to 1 and not 28? (e.g., *Since there are no brackets, you multiply 2 × 4 first and then subtract from 9.*)

You will need

- Student Resource pages 17–19

Using the Guided Intervention Student Resource pages 17–19

Read about the skill-testing question and BEDMAS together. Then work through the prompts to complete the calculation together. Have students who need calculators use them for individual calculations, not the entire expression, since some calculators use the order of operations.

There is no mention of the E (for exponents) in BEDMAS in the Student Resource. However, you might let students know that later they will use special symbols called exponents, involving repeated multiplication.

Have students work through the **Try These** questions in pairs or individually.

Observe whether students

- can apply the order of operations (Questions 1, 4, 5, 6, 7, 8)
- can communicate how to perform a series of calculations (Questions 2, 7)
- recognize when there are options in evaluating an expression (Question 3)
- can insert brackets into an expression to achieve a given result (Question 8)
- recognize why the order of operations rules are needed (Questions 6, 7, 9, 10)

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- Why is there a choice about which calculation to perform first in Question 1b)? (e.g., *If there is more than one pair of brackets, it doesn't matter which you do first.*)
- Why were some answers to Question 5 the same? (e.g., *For parts e) and g), if you divide and then multiply, you get the same result as multiplying and then dividing.*)
- What strategy did you use in Question 8 to decide where to place the brackets? (e.g., *For part b), I created 2 numbers that, when divided, had a quotient of 5;* $4 \times 30 + 5 = 125$ *and* $17 + 8 = 25$ *and* $125 \div 25 = 5$.)
- Are there situations where people would likely do the correct order of operations even if they didn't know the rules? (*Yes.* e.g., *If all of the operations were the same, you would get the right answer anyway.*)
- Why are the order of operations rules more important when expressions are written rather than said? (e.g., *When you say an expression like 8 + 3 × 2, you can say, "Add 8 to the product of 3 × 2" or "Multiply 3 by 2 first and then add 8."*)

Pathway 2
OPEN-ENDED

Dividing Whole Numbers

You will need

- base ten blocks (optional)
- Student Resource pages 20–21

Open-Ended Intervention

Before Using the Open-Ended Intervention

Tell students that 4 friends shared $84. Ask:

- How would you figure out how much each gets?
  (e.g., *If they each got $20 that would be $80, so they can each get $1 more.*)
- There are 84 books to be put in packages of 4. How would you figure out the number of packages? Why? (e.g., *I'd divide 84 by 4. I'm making a lot of equal packages, or groups, and you divide to find out how many equal groups.*)
- What is 84 ÷ 4? How did you get your answer?
  (*21.* e.g., *There are 4 groups of 20 in 80, and then there is 1 more group of 4.*)
- Why were the answers the same when you shared $84 and packaged 84 books?
  (e.g., *Both times it was about how many 4s are in 84.*)

Using the Open-Ended Intervention **Student Resource pages 20–21**

Read through the tasks on the student page together. You might provide base ten blocks. If students seem unfamiliar with how to model, you might refer them to the instructional part of the Guided intervention. Give students time to work, ideally in pairs.

Observe whether students

- can estimate quotients
- relate a division statement to a problem situation
- can divide three-digit or four-digit numbers by one-digit numbers
- use different strategies to divide
- relate multiplication to division

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- Why could you solve the problems by multiplying or dividing?
  (e.g., *You can divide when forming equal groups from a total amount or you can multiply to figure out if you have the total you want.*)
- How did you check your answers?
  (e.g., *I multiplied my answer by the team size and added the remainder to see if I got the right number of students.*)
- Which calculations did you find easiest to do? Why?
  (e.g., *dividing by 5, since I divided by 10 and then multiplied by 2*)
- What strategies did you use to solve the team size problems?
  (e.g., *I usually broke up the number into parts that were easy to divide by the team size. Sometimes I used base ten blocks.*)
- What did you decide to do with "leftover" students?
  (e.g., *It depended on how many were left over. If it was a small number, I made a couple of teams a bit bigger. For a bigger number, I made another, smaller team.*)

Dividing Whole Numbers

Pathway 2 GUIDED

Guided Intervention

You will need

- base ten blocks (optional)
- Student Resource pages 22–25

Before Using the Guided Intervention

Tell students that 4 friends shared $84. Ask:

- ▸ How would you figure out how much each gets?
  (e.g., *If they each got $20 that would be $80, so they can each get $1 more.*)
- ▸ There are 84 books to be put in packages of 4. How would you figure out the number of packages? Why?
  (e.g., *I'd divide 84 by 4. I'm making a lot of equal packages, or groups, and you divide to find out how many equal groups.*)
- ▸ What is 84 ÷ 4? How did you get your answer?
  (*21.* e.g., *There are 4 groups of 20 in 80, and then there is 1 more group of 4.*)
- ▸ Why were the answers the same when you shared $84 and packaged 84 books?
  (e.g., *Both times it was about how many 4s are in 84*).

Using the Guided Intervention **Student Resource pages 22–25**

Work through the instructional section together, having students model and complete each strategy for each type of division (sharing and grouping). You might provide base ten blocks.

Have students work through the **Try These** questions in pairs or individually.

Observe whether students

- can estimate quotients (Questions 1, 3, 9)
- can model division calculations (Question 2)
- use different strategies to divide (Questions 2, 5, 8)
- relate division to problem situations (Question 4)
- can calculate quotients (Questions 6, 7)
- relate multiplication to division (Question 10)

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- ▸ How did you estimate 3021 ÷ 7 in Question 3?
  (e.g., *I divided 2800 ÷ 7 = 400, since 28 is easy to divide by 7 and 2800 is close to 3021.*)
- ▸ What did the problem situations you created for Question 4 have in common?
  (e.g., *They were both about taking an amount and making equal groups.*)
- ▸ What strategies did you use to divide in Question 6?
  (e.g., *I usually broke up the number I was dividing into parts that are easy to divide by the second number. Sometimes I used base ten blocks.*)
- ▸ How could you use multiplication to help you answer Question 7?
  (e.g., *For part a), you could multiply 83 by one-digit numbers to get a product greater than 500.*)

Pathway 3
OPEN-ENDED

Multiplying Whole Numbers

You will need

- empty egg cartons
- base ten blocks (optional)
- Student Resource pages 26–27

Open-Ended Intervention

Before Using the Open-Ended Intervention

Show students an empty egg carton and establish that it holds 12 eggs. Tell them to imagine that there are 9 full cartons. Ask:

- How would you figure out how many eggs there would be, without counting every one?
  (e.g., *I'd multiply. I know* $9 \times 10 = 90$ *and* $9 \times 2 = 18$*, so I'd add* $90 + 18$*.*)
- How would you figure out the number of eggs in 21 cartons?
  (e.g., *There are 120 eggs in 10 cartons, 120 eggs in the next 10, and 12 more. I'd add* $120 + 120 + 12$*.*)
- What models might help you multiply 21 by 12?
  (e.g., *I'd use base ten blocks and show 21 groups of 12, using 21 ten blocks and 21 pairs of one blocks.*)

Using the Open-Ended Intervention **Student Resource pages 26–27**

Read through the tasks on the student pages together. You might provide base ten blocks. If students seem unfamiliar with how to model multiplication, you might refer them to the instructional part of the Guided Intervention. Give students time to work, ideally in pairs.

Observe whether students

- can estimate products
- can calculate products of 2 two-digit numbers
- relate multiplication to a problem situation
- use different strategies to multiply

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- Why did you choose to solve the problems by multiplying?
  (e.g., *You multiply when you are counting the total of a lot of equal groups.*)
- What strategies did you use to multiply?
  (e.g., *I made a base ten block rectangle with one number as the length and one number as the width, and counted the blocks. I also broke the number into smaller parts and multiplied each part and then added them together.*)
- How did you check to see if your answers were reasonable?
  (e.g., *I estimated by using close-by friendly numbers,* e.g., 38×37 *is almost* $40 \times 40 = 1600$*, so I knew my answer had to be less than but close to 1600.*)

Multiplying Whole Numbers

Pathway 3 GUIDED

Guided Intervention

You will need

- empty egg cartons
- base ten blocks
- Student Resource pages 28–31

Before Using the Guided Intervention

Show students an empty egg carton and establish that it holds 12 eggs. Tell them to imagine that there are 9 full cartons. Ask:

- How would you figure out how many eggs there would be, without counting every one?
  (e.g., *I'd multiply. I know* $9 \times 10 = 90$ *and* $9 \times 2 = 18$*, so I'd add* $90 + 18$.)
- How would you figure out the number of eggs in 21 cartons?
  (e.g., *There are 120 eggs in 10 cartons, 120 eggs in the next 10, and 12 more. I'd add* $120 + 120 + 12$.)

Using the Guided Intervention **Student Resource pages 28–31**

Read the opening paragraph. Provide base ten blocks and work through the instructional section together, having students model and complete each strategy.

Have students work through the **Try These** questions in pairs or individually.

Observe whether students

- can estimate products (Questions 1, 3, 9)
- can model multiplication (Question 2)
- relate multiplication to problem situations (Question 4)
- use different strategies to multiply (Questions 5, 7)
- can calculate products (Questions 5, 6, 7, 8, 9, 10)
- recognize computational errors (Question 7)
- relate 2 products (Question 8)

Consolidating and Reflecting

Ensure understanding by asking questions such as these based on students' work:

- How did you estimate 35×49 in Question 3?
  (e.g., *I figured it was about halfway between* $30 \times 50 = 1500$ *and* $40 \times 50 = 2000$.)
- What sort of problem situations did you create for Question 4?
  (e.g., *I created problems where there were lots of items in equal groups and I needed to know the total.*)
- What model could you create to show Ling why she was wrong in Question 7?
  (e.g., *I'd create a base ten block rectangle, and she would see that the 4 parts of the area have to be added.*)
- What strategies did you use to multiply in Question 8?
  (e.g., *I broke the numbers into parts, multiplied the parts, and added them together. I also made base ten block rectangles.*)

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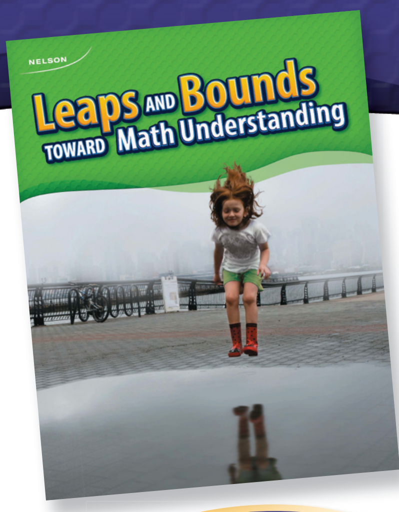

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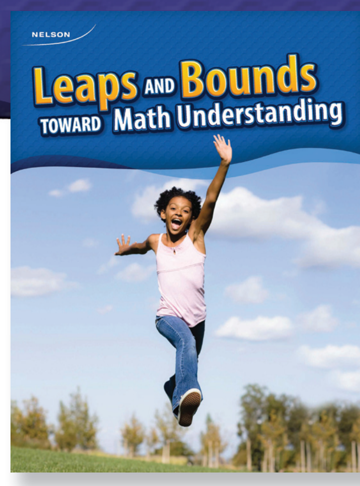

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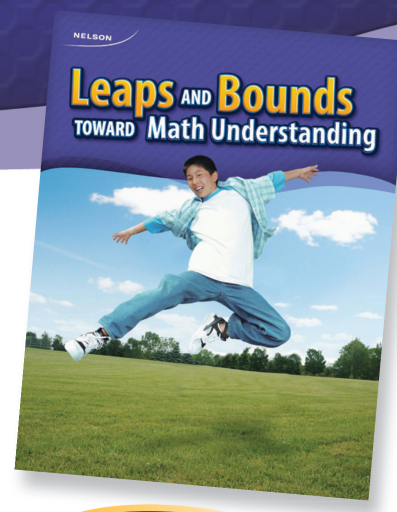

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01/12

ISBN-13: 978-0-17-665622-5
ISBN-10: 0-17-665622-7

9 780176 656225

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