

Writing Pattern Rules

Goal Use rules to extend patterns and write pattern rules.

1. Steve learned that a year on Venus is about seven Earth months long.

- a) Complete the table for Venus.

Venus	
Number of years on Venus	Number of months on Earth
1	7
2	14
3	21
4	28

- b) Write a pattern rule to calculate the number of Earth months in any number of years on Venus.

Multiply 7 by the term number.

- c) Use your pattern rule to calculate the approximate number of Earth months in 12 years on Venus.

about 84 months

$$7 \times 12 = 84$$

2. Bev saves \$12 each month from her paper route.

- a) Write a pattern of numbers that shows the amount Bev saves in 1 to 4 months.

Number of months	Amount
1	\$12
2	\$24
3	\$36
4	\$48

- b) Write a pattern rule to calculate the amount she saves in any number of months.

Multiply \$12 by the term number.

- c) Use your pattern rule to calculate the amount Bev saves in 10 months.

$$\$12 \times 10 = \$120$$

- d) Bev wants to buy a new hockey jersey for \$100. For how many months does she need to save?

about 9 months $\$12 \times 9 = \108

Bev will have \$108 after 9 months.

At-Home Help

A **pattern** is a group of numbers, shapes, or objects that follow a rule while repeating or changing.

To extend a pattern you can use a table or a pattern rule that relates the term number to the pattern rule.

A **term number** is the number that tells the position of an item in a pattern.

For example, the pattern 2, 4, 6, 8, 10, ... can be shown in a table like this:

Term number	Number in pattern
1	2
2	4
3	6
4	8
5	10

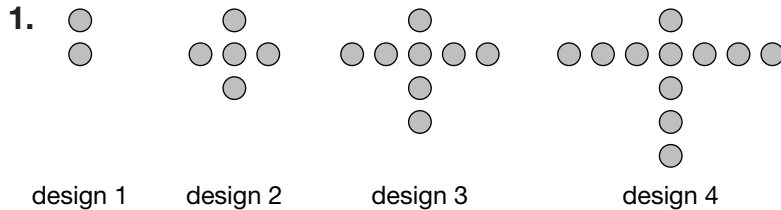
A pattern rule to get any number in the pattern is multiply 2 by the term number.

$$10\text{th term} = 2 \times 10$$

$$= 20$$

Relationship Rules for Patterns

Goal Write relationship pattern rules based on the term number.



a) Complete the table to show the number of dots in designs 1 to 4.

Design number	Number of dots
1	2
2	5
3	8
4	11

b) Write the first term and the common difference.

The first term is 2. The common difference is 3.

c) How many dots are in design 8?

23 dots

8th term = 2 + seven 3s

= 2 + 21

= 23

2. Determine the 11th term in each pattern. Use a pattern rule. Show your work.

a) 1, 5, 9, 13, ...

The first term is 1. The common difference is 4.

11th term = 1 + ten 4s

= 1 + 40

= 41

b) 21, 26, 31, 36, ...

The first term is 21. The common difference is 5.

11th term = 21 + ten 5s

= 21 + 50

= 71

c) 2.2, 4.4, 6.6, 8.8, ...

The first term is 2.2. The common difference is 2.2.

11th term = 2.2 + ten 2.2s

= 2.2 + 22

= 24.2

d) \$1.25, \$1.75, \$2.25, \$2.75, ...

The first term is \$1.25.

The common difference

is \$0.50. 11th term = \$1.25 + ten \$0.50s

= \$1.25 + \$5.00

= \$6.25

At-Home Help

A **recursive pattern rule** is a pattern rule that tells you the start number of a pattern and how the pattern continues.

For example, a recursive rule for the pattern 5, 8, 11, 14, ... is start with 5 and add 3.

A **common difference** is the difference between any two consecutive terms in a pattern. Not all patterns have a common difference.

For example, the pattern

5, 10, 15, 20, ...


has a common difference of 5.

The pattern

1, 4, 9, 16, ...


has no common difference.

An **explicit pattern rule** is a pattern rule that tells you how to get any term in the pattern without listing all the terms before it.

For example, an explicit pattern rule for 5, 8, 11, 14, ... uses the first term (5) and the common difference (3).

To calculate the 20th term,
 20th term = first term + [(term number - 1) × (common difference)]
 = 5 + (19 × 3)
 = 5 + 57
 = 62

Variables in Expressions

Goal Use variables in an expression.

1. Arpita is baking cookies for a school bake sale. One batch of cookies uses 75 g of chocolate chips.

- a) Calculate the number of grams of chocolate chips in the first four batches of cookies.

Suggested answer:

Batch number	Number of grams of chocolate chips
1	75
2	150
3	225
4	300

- b) Write an explicit pattern rule for the number of grams of chocolate chips in the 10th batch.

10th term = 75×10

- c) Write your pattern rule using a variable for the batch number. Use the variable b .

$75 \times b$

- d) Calculate the number of grams of chocolate chips in the first four batches using your answer in part c). Show your work.

Suggested answer: for $b = 1$, $75 \times 1 = 75$ g

for $b = 2$, $75 \times 2 = 150$ g

for $b = 3$, $75 \times 3 = 225$ g

for $b = 4$, $75 \times 4 = 300$ g

- e) How many grams of chocolate chips does Arpita need to make 11 batches of cookies?

825 g

Suggested answer: for $b = 11$, $75 \times 11 = 825$

At-Home Help

A **variable** is a letter or symbol that is used to show a quantity. This quantity can have different values.

For example, t is a variable that could be used to represent the amount of time you surf the Internet each day.

Variables are usually used when writing explicit pattern rules to make the rules easier to write.

For example, an explicit pattern rule for the pattern 50, 100, 150, 200, ... is $50 \times n$. The variable n is the term number.

Term (n)	Value ($50 \times n$)
1	$50 \times 1 = 50$
2	$50 \times 2 = 100$
3	$50 \times 3 = 150$

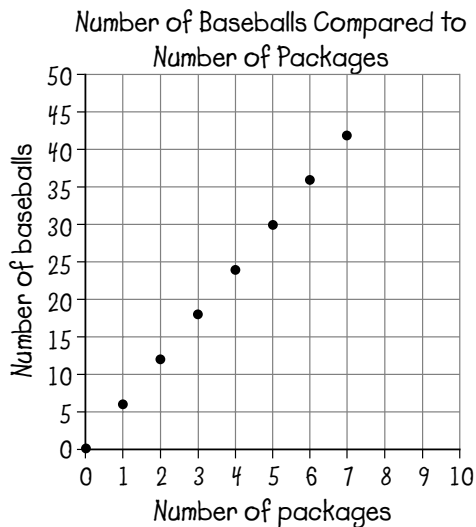
Representing Patterns on a Graph

Goal Represent patterns in tables and on graphs.

1. George buys baseballs in packages of 6.
- a) Complete the table to show the total number of baseballs in 0 to 6 packages.

Number of packages	Number of baseballs
0	0
1	6
2	12
3	18
4	24
5	30
6	36

- b) Graph the number of baseballs compared to the number of packages.



- c) Write a pattern rule to calculate the number of baseballs in any number of packages. Use the variable n in your rule.
- $6 \times n$
- d) Determine the number of baseballs in 15 packages.
- $6 \times 15 = 90$
There are 90 baseballs in 15 packages.

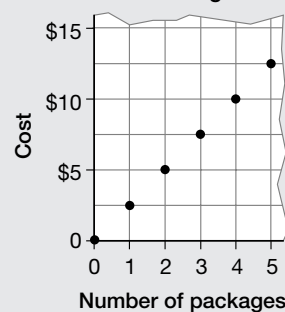
At-Home Help

Patterns can be represented in tables or graphs.

For example: It costs \$2.50 to buy a package of juice boxes. To determine the cost of 20 packages, you can extend a table or graph, or use a pattern rule.

Number of packages	Cost
0	0
1	\$2.50
2	\$5.00
3	\$7.50
4	\$10.00
5	\$12.50

Cost Compared to Number
of Packages



A pattern rule to calculate the cost for any number of packages is $\$2.50 \times n$.

$$\begin{aligned} \text{Cost of 20 packages} &= \$2.50 \times 20 \\ &= \$50.00 \end{aligned}$$

Patterns and Spreadsheets

Goal Create patterns using spreadsheets and compare the growth.

Pamela wants to know how much money she would have if her bank tripled her money every day for 30 days. She starts with \$2.

1. Complete the table or spreadsheet to show a pattern for Pamela's money. *Suggested answer:*

	A	B
1	Day number	Amount
2	1	\$2
3	2	\$6
4	3	\$18
5	4	\$54

2. What is the formula for calculating the amount on day 4?

Suggested answer: $\text{amount} = 3 \times \18 or $B5 = 3 \times B4$

3. a) Predict the amount Pamela would have on day 14.

Suggested answer:

I predict that she would have \$1 000 000.

- b) Calculate the amount on day 14.

\$3 188 646

- c) How close was your prediction?

Suggested answer: My prediction was too low. Pamela would get over \$3 000 000 on day 14.

4. Describe the pattern in the amounts Pamela would have.

Suggested answer: The first term in the pattern is \$2. This pattern does not have a common difference. To get any term in the pattern, you have to multiply the term before it by 3.

At-Home Help

Patterns can be represented using a spreadsheet. A **spreadsheet** is a computer program that has columns of data that are related. Each number in a spreadsheet has its own cell.

To represent a pattern, enter information for the first term. Use one or more operations to get the rest of the terms in the pattern.

For example, the spreadsheet below shows a pattern. The first term is \$2.

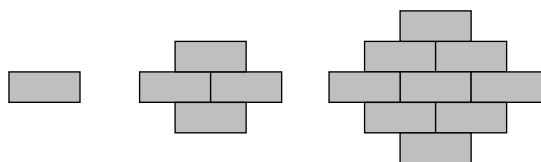
	A	B
1	Term number	Cost
2	1	\$2
3	2	\$4
4	3	\$8
5	4	\$16

The formula to get the second term is $B3 = 2 \times B2$, the third term is $B4 = 2 \times B3$, and so on.

Solve a Simpler Problem

Goal Solve problems by using a simpler problem.

1. Use this pattern.



design 1

design 2

design 3

a) How many boxes are in design 8? Make a plan to solve the problem.

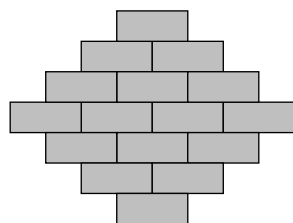
Suggested answer: I don't want to draw design 8, so I'll solve a simpler problem. I'll draw design 4.

I'll make a table to compare the design number with the number of boxes.

I'll look for a pattern in the numbers in the table and see if the number of boxes for design 4 matches my drawing. Then I'll extend the table to design 8.

Carry out your plan.

Suggested answer:



design 4

Design number	Number of boxes
1	1
2	4
3	9
4	16

I notice that if I multiply the design number by itself, I get the number of boxes.

$$1 \times 1 = 1$$

$$2 \times 2 = 4$$

$$3 \times 3 = 9$$

$$4 \times 4 = 16$$

For design 8, I predict that there will be $8 \times 8 = 64$ boxes.

b) How many boxes are in design 10?

Suggested answer:

$$10 \times 10 = 100$$

100 boxes

At-Home Help

To solve some problems, it is easier to solve a simpler problem.

Make a Plan

Organize data using the simpler problem. If possible, use a table to arrange numbers and drawings.

Carry Out the Plan

Look for a pattern to relate the columns in your table.

Try to find an operation that works for all the rows in your table.

Write a pattern rule.

Check that your pattern rule works for the next simple problem. Draw a picture, if necessary, to check.

Use the pattern rule to solve the original problem.

Equal Expressions

Goal

Write equal expressions and determine the value of a missing term in an equation.

1. Which expressions are equal? Use an equals sign. If they are not equal, change one expression to make them equal.

a) $2 + 3 \square = 5 + 0$

b) $1 + 7 \square 2 + 5$

Suggested answer: $0 + 7 = 2 + 5$

c) $8 + 6 \square 2 + 9$

Suggested answer: $8 + 6 = 5 + 9$

d) $3 + 8 \square = 6 + 5$

2. Replace each \square so the expressions are equal.

a) $5 + \square = 7 + 6$

$\square = 8$

c) $\square \times 3 = 2 + 7$

$\square = 3$

b) $9 - 4 = \square + 3$

$\square = 2$

d) $12 \div \square = 4 \times 3$

$\square = 1$

3. Isabelle has five swim practices and three soccer practices this month. Judy has the same number of practices this month. Judy has two swim practices.

- a) Write an expression for the number of practices Isabelle has.

$5 + 3$

- b) Write an expression for the number of practices Judy has.

$2 + p$

- c) Write an equation with your expressions.

$5 + 3 = 2 + p$

- d) How many soccer practices does Judy have? Explain what you did.

Suggested answer: I know that $5 + 3 = 8$. So $2 + p = 8$. This means p is 6. Judy has 6 soccer practices. An expression for the number of practices Judy has is $2 + 6$. $5 + 3 = 2 + 6$ Both expressions have the same value.

At-Home Help

An **expression** is a mathematical statement made with numbers or variables and operations.

For example: $5 + 3 - 7$ is an expression with numbers only.

$6 \times t$ is an expression with a variable.

An **equation** is a mathematical statement that has an expression on each side of the equal sign. Both expressions must be equal in an equation.

Variables in Equations

Goal Solve equations including symbols representing variables.

1. A gym has twice as many soccer balls as basketballs. Altogether there are 36 balls.

- a) Explain what is represented by the equation $S + B = 36$.

Suggested answer: It represents the total number of balls in the gym. S is the number of soccer balls. B is the number of basketballs. There are 36 balls.

- b) Explain what is represented by the equation $S = B + B$.

Suggested answer: It represents the relationship between the number of soccer balls and the number of basketballs. The number of soccer balls is equal to two times the number of basketballs.

- c) How many basketballs are there? How many soccer balls are there?

Suggested answer:

$S + B = 36$ $S = B + B$, so I can replace S with $B + B$.

$$B + B + B = 36$$

$$12 + 12 + 12 = 36$$

$B = 12$ and $S = 24$ There are 12 basketballs and 24 soccer balls.

2. In a granola recipe, there is three times as much oats as coconut. The total mass of oats and coconut is 600 g.

- a) Explain what is represented by the equation $O + C = 600$.

Suggested answer: It represents the total mass. O represents the mass of oats.

C represents the mass of coconut. The total mass is 600 g.

- b) Explain what is represented by the equation $O = C + C + C$.

Suggested answer: It represents the relationship between the mass of oats and the mass of coconut. The mass of oats is equal to three times the mass of coconut.

- c) How many grams of each ingredient are there?

Suggested answer:

$O + C = 600$ I can replace O with $C + C + C$.

$$C + C + C + C = 600$$

I know $4 \times 150 = 600$, so there are 150 g of coconut and $O = C + C + C$ so there are 450 g of oats.

At-Home Help

An equation can sometimes have more than one variable.

For example, $A + O = 9$ and $A = O + O$ are equations with the variables A and O .

To solve for A and O , use the expression $O + O$ for A in the first equation.

$$O + O + O = 9$$

Since three 3s are 9, O must be 3.

Use 3 for O in the second equation to get $A = 3 + 3$. Since $3 + 3 = 6$, A must be 6.

Test Yourself Page 1

Circle the correct answer.

1. One movie ticket costs \$5.50. Which table shows the correct pattern?

A.

Number of tickets	Cost
1	\$5.50
2	\$10.00
3	\$15.50
4	\$20.00

C.

Number of tickets	Cost
1	\$5.50
2	\$6.50
3	\$7.50
4	\$8.50

B.

Number of tickets	Cost
1	\$5.50
2	\$11.00
3	\$16.50
4	\$22.00

D.

Number of tickets	Cost
1	\$0
2	\$5.50
3	\$11.50
4	\$16.50

2. Which pattern rule best represents the pattern in Question 1?

A. Add \$5.50 to the term number.

C. Multiply \$5.50 by the term number.

B. Multiply \$5.50 by 2.

D. Add 1 to \$5.50.

3. What is the cost of six movie tickets in Question 1?

A. \$27.50

C. \$33.00

B. \$30.00

D. \$33.50

4. Which pattern rule shows the total number of candies in any number of packages?

Number of packages	Total number of candies
1	3
2	6
3	9
4	12
5	15

A. $2 \times n$

B. $3 \times n$

C. $2 + n$

D. $3 + n$

5. What is the common difference in the pattern 18, 21, 24, 27, ...?

A. 2

C. 4

B. 3

D. 5

Test Yourself Page 2

6. What is the first term and the common difference in the pattern \$1.50, \$3.00, \$4.50, \$6.00, ...?
- A. \$1.50, \$1.50 C. \$3.00, \$1.50
B. \$1.50, \$3.00 D. \$1.50, \$2.00
7. What is the 10th term in the pattern in Question 6?
- A. \$12.00 C. \$14.00
B. \$13.00 D. \$15.00
8. Which expressions are *not* equal?
- A. $2 + 8$ and $6 + 4$ C. 3×4 and $8 + 2$
B. $9 - 5$ and 2×2 D. $7 + 1$ and $10 - 2$
9. Tilo has two red baseball caps and five green baseball caps. Michael has the same total number of baseball caps as Tilo. Michael has three green baseball caps. Which equation would you use to solve this problem?
- A. $3 + 5 = 2 + c$ C. $2 + c = 5 + 3$
B. $2 + 3 = 5 + c$ D. $2 + 5 = c + 3$
10. How many red baseball caps does Michael have in Question 9?
- A. 2 C. 4
B. 3 D. 5
11. A closet has three times as many hats as sweaters. The total number of hats and sweaters is 12. How many of each item is there?
- A. 8 hats, 4 sweaters C. 7 hats, 5 sweaters
B. 9 hats, 3 sweaters D. 6 hats, 6 sweaters