

## ALGEBRA

The Number Sense and Numeration strand and the Patterning and Algebra strand of the Ontario Curriculum for Grade 8 identify seven Mathematical Process Expectations: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating.
Using these process expectations, students have studied, learned, and applied Grade 8 concepts and skills organized under the headings of

- Number Sense and Numeration
- Quantity Relationships
- Operational Sense
- Patterning and Algebra
- Patterns and Relationships
- Variables, Expressions, and Equations

In Grade 9, students will study, learn, and apply concepts and skills organized under the headings of

- Number Sense and Algebra
- Operating with Exponents
- Manipulating Expressions and Solving Equations

The following chart highlights key knowledge and skill development as students make the transition from Grade 8 to Grade 9 Principles of Mathematics (Academic).

GRADE 8 EXPECTATIONS
GRADE 9 EXPECTATIONS

- describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by $x$; area, which is two dimensional, can be represented by $(x)(x)$ or $x^{2}$; volume, which is three dimensional, can be represented by $(x)(x)(x),\left(x^{2}\right)(x)$, or $\left.x^{3}\right]$
- express repeated multiplication using exponential notation (e.g., $2 \times 2 \times 2 \times 2=2^{4}$ )
- derive, through the investigation and examination of patterns, the exponent rules for multiplying and dividing monomials, and apply these rules in expressions involving one and two variables with positive exponents
- extend the multiplication rule to derive and understand the power of a power rule, and apply it to simplify expressions involving one and two variables with positive exponents


## GRADE 9 EXPECTATIONS

- evaluate expressions that involve integers, including expressions that contain brackets and exponents, using order of operations
- evaluate algebraic expressions with up to three terms, by substituting fractions, decimals, and integers for the variables (e.g., evaluate $3 x+4 y=2 z$, where $x=\frac{1}{2}, y=0.6$, and $z=-1$ )
- substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^{3}$ by hand and $9.8^{3}$ by using a calculator])
- add and subtract polynomials with up to two variables [e.g., $(2 x-5)+(3 x+1),\left(3 x^{2} y+2 x y^{2}\right)+$ $\left(4 x^{2} y-6 x y^{2}\right)$, using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil)
- multiply a polynomial by a monomial involving the same variable [e.g., $2 x(x+4), 2 x^{2}\left(3 x^{2}-2 x+1\right)$ ], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil)
- expand and simplify polynomial expressions involving one variable [e.g., $2 x(4 x+1)-3 x(x+2)$ ], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil)
- solve first-degree equations, including equations with fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies (e.g., the balance analogy, algebraic strategies)
- rearrange formulas involving variables in the first degree, with and without substitution (e.g., in analytic geometry, in measurement) one-variable term and having solutions that are integers, by using inspection, guess and check, and a "balance" model
- describe different ways in which algebra can be used in real-life situations (e.g., the value of $\$ 5$ bills and toonies placed in an envelope for fund raising can be represented by the equation $v=5 f+2 t$ )
- translate statements describing mathematical relationships into algebraic expressions and equations (e.g., for a collection of triangles, the total number of sides is equal to three times the number of triangles or $s=3 n$ )
- solve problems that can be modelled with first-degree equations, and compare algebraic methods to other solution methods

For each strand of the curriculum, or part of a strand, ONAP 9 provides two types of assessment materials. Consider the following points when administering the assessments for this strand.

## Part A: Activating Prior Knowledge (page 72)

- The activities in Part A have been created to activate students' knowledge before they complete the Part B or C assessment.
- No score is assigned during Part A.
- One or both of the activities in Part A can be used in a variety of ways, such as
- for warm-up at the beginning of a period
- at the end of a period when time allows
- for a full period
- as a game or an activity to take home
- as an activity to revisit for short amounts of time over several periods


## Part B: Concepts and Skills Assessment (page 83)

- The assessment in Part B addresses specific expectations within two of the three overall expectations in the Grade 8 Number Sense and Numeration strand and within the two overall expectations in the Grade 8 Patterning and Algebra strand.
- Students will be responding to a mix of questions: short response, explanation, fill-in-the-blank, and multiple choice.


## Part C: Performance-Based Assessment

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- The performance task in Part C is designed to provide insights into how well students are able to perform in terms of the categories of the Ontario Achievement Chart: Knowledge and Understanding, Thinking, Communication, and Application.
- Overall expectations in the Grade 8 Number Sense and Numeration strand and in the Grade 8 Patterning and Algebra strand that relate to the Algebra part of the Number Sense and Algebra strand of Grade 9 Principles of Mathematics (Academic) are assessed through the Concepts and Skills Assessment in Part B.


## Part A: Activating Prior Knowledge

## Administration

To activate students' knowledge of the Number Sense and Numeration strand and the Patterning and Algebra strand from Grade 8, choose one or both of the following activities prior to administering the Part B or C assessment. Introductory and culminating suggestions are provided for each activity. No score is assigned for these activities.

## Timing

One or both of the activities can be used in a variety of ways, such as

- for warm-up at the beginning of a period
- at the end of a period when time allows
- for a full period
- as a game or an activity to take home
- as an activity to revisit for short amounts of time over several periods


## Accommodating Students with Special Needs

Observe students as they complete the activities. While the activities in this section are not designed as a formal diagnostic assessment, you may want to consider whether students who are having significant difficulties with the activities are ready to participate in the full ONAP assessment for this grade level. Observations at this stage might also indicate students who will need special accommodations during the assessment, such as having someone read questions to them or scribe responses. Some students may benefit from completing the assessment in a resource room.

## Activity 1: Modelling Relationships from Patterns

This activity addresses representing a pattern in different ways: modelling using materials, using a table of values, describing the pattern in words, and writing an algebraic expression.

## Materials

- materials for modelling patterns, such as counters, square tiles, linking cubes, and toothpicks
- pencils and paper


## Introducing the Activity

Present students with the following models:



Figure 1


Figure 2


Figure 3


Ask students to consider the following, leading them to discuss how the different models represent the same pattern.
a) How are these representations the same?
b) How are they different?
c) Describe the patterns you see.

Start a table of values to represent the relationship between the figure number and the number of objects based on the first three figures in each representation. Ask students to predict the number of objects needed for Figures 4, 5, and 6.

| Figure number | Number of objects |
| :---: | :---: |
| 1 | 8 |
| 2 | 13 |
| 3 | 18 |
| 4 | 23 |
| 5 | 28 |
| 6 | 33 |

Discuss different ways to describe the patterns in words. For example:

- The value starts with 5 plus 3, and increases by 5 each time.
- There are groups of 5 and 3 more. The number of groups of 5 in each figure increases by one group from one figure to the next, so the number of objects in each figure increases by 5 each time.

Discuss how the pattern rule written in words can be simplified using variables.

- The number of groups of 5 in each figure is the variable n , so 5 is the coefficient for n . The value of n is the same as the figure number. So, to represent the number of objects in a figure, I can use the algebraic expression $5 \mathrm{n}+3$.

Guide students through evaluating the expression, substituting different values for $n$. For example, when $n=2,5 n+3=5(2)+3=13$; when $n=3,5 n+3=5(3)+3=18$. Emphasize that the models, word description, table of values, and algebraic expression represent the same relationship.

Ask students how they can predict the number of objects needed to represent the 20th figure. Include in the discussion continuing the model to the 20th figure, extending the table of values to the 20th figure, or substituting 20 for $n$ and evaluating the expression. Discuss which of these strategies is the most efficient and why.

## Procedure

Have students work in groups of two or three, or individually, according to available materials, student preferences, and student abilities. If possible, provide groups or individual students with different materials for modelling algebraic expressions.
Tell students to create their own growing pattern from the materials. Stipulate that the pattern must grow by the same amount each time. Ask students to use their patterns to complete the following:

1. Create a table of values that shows the relationship between each figure number (or term number) and the number of objects (or term value).
2. Describe the pattern rule in words.
3. Describe the pattern rule using an algebraic expression.

Circulate around the room and ask students the following questions:
a) What does the variable represent in your expression?
b) How does your expression relate to the pattern rule you described in words?
c) How many objects would there be in the 50th figure in your pattern?

Then ask students to create a pattern that shrinks by the same amount each time, and repeat parts 1 through 3 above. Have them answer questions a) to c). Have students keep models for reference during the Culminating Discussion.

## Culminating Discussion

1. How did you represent your pattern?

- I used counters because I could move them around as I figured out how to represent the pattern I wanted.
- I drew a diagram because it was easier and faster.
- I started with toothpicks for the first two terms because it was easier to change them. Then I drew a diagram for another term or two because the toothpicks became difficult to work with.

2. How did you decide on the strategy to figure out the value of a term?

- I used counters for values up to the 4th term because it was clearer. For terms such as the 5th or 6th, I extended the pattern in the table of values by adding if it was a growing pattern, or subtracting if it was a shrinking pattern. For the 50th term, I substituted into the algebraic expression because it was faster.
- Sometimes I used a table of values because it showed the pattern. At other times, I used the word description because it made sense to me.
- I always substituted into the algebraic expression because I found it to be the most efficient method.

3. What did this activity show you about representing algebraic expressions?

- It reminded me of the different ways I learned to represent an algebraic expression.
- It showed how different models can represent one expression.
- It showed that some strategies for representing an algebraic expression are clearer for the first terms, but other strategies are better for later terms.
- It proved you can represent an algebraic expression in one way to figure out a term, and then use a different way to check.


## Activity 2: Four Puzzles

This activity requires matching equations with solutions to the equations.

## Materials

- electronic display board or overhead projector (optional)
- BLM A1: Activity 2: Four Puzzles (one for each group of 4 students)
- BLM A2: Activity 2: Four Puzzles Cards (9 cards from a different page per student in each group of 4 students)
- pencils and paper


## Introducing the Activity

Initiate a discussion about solving equations by presenting an equation such as $2 x-1=5$. Ask: What does it mean to solve an equation? Elicit from students that solving an equation means determining a value for the variable that results in the same value for the left and right sides of the equation. You might have students work briefly with partners to discuss their solution strategies for $2 x-1=5$ before discussing the solution as a class. Invite a few pairs to share their strategies with the class or in groups. Encourage discussion about a variety of strategies, such as the following. If necessary, make suggestions about these strategies.

- Think about balancing the sides. The right side is 5 . To get 5 on the left side, use a number that when you subtract 1 from it, the result is 5 . Since 6 minus 1 equals 5 , $2 x$ has to be 6 . That means that $x$ must be 3 since 2 times 3 is 6 .
- Use guess and check to try values for $x$, calculating the value of $2 x-1$ until the result is 5 . For example, students might first try $x=1$ to get $2(1)-1=2-1$ or 1 , which is too low. They might then try $x=2$ to get $2(2)-1=4-1$ or 3 , which is still too low, and follow that by trying $x=3$ to get $2(3)-1=6-1$ or 5 , which equals the right side. So $x=3$.
- Add 1 to $2 x-1$ to leave only the term with the variable, $2 x$, and then add 1 to 5 to balance both sides of the equation. That gives $2 x-1+1=5+1$, or $2 x=6$. You need just $x$ on one side. You can get just $x$ on one side by dividing $2 x$ by 2 because $2 x \div 2=x$. You need to divide the other side of the equation by 2 because you need to balance both sides. Since $6 \div 2=3, x$ must be 3 .

Ask: How can you decide if the solution is correct? Elicit from students that one side of the equation must balance the other side, so that when the solution is substituted for $x$ and the value of each side is calculated, the result is the same for each side. For $2 x-1=5$, guide students with substituting 3 for $x$ in the left side $2 x-1$ to get $2(3)-1=5$, which is the value of the right side. Discuss how the equal sign means that the value of one side of an equation is equal to the value of the other side.
Use a similar procedure for equations such as $6 x \div 2=-18$ and $5 x+6=-14$. Emphasize the need to balance both sides of the equation.

## Procedure

Distribute one copy of BLM A1: Activity 2: Four Puzzles to each group of students. Next, give each student in the group a different set of nine puzzle pieces cut from BLM A2: Activity 2: Four Puzzles Cards. Discuss the instructions outlined on BLM A1. You might have a copy of BLM A1 displayed for this discussion. Ensure students understand that each student in the group fits together the nine cards as one square puzzle, and then the group works as a team to fit the four individual puzzles together to form a larger square puzzle.
If some students in a group finish their individual puzzles before other team members, encourage students to help one another. The variation described on BLM A1: Activity 2: Four Puzzles provides a suggestion for an individual activity as an alternative, or at a later time.

The following is the arrangement of the four puzzles for the whole puzzle.


## Culminating Discussion

1. What strategies did you use to match equations with solutions?

- Sometimes I solved an equation first, and then I looked for a card with that solution. At other times, I started with a solution and used guess and check to find an equation to go with it.
- If I wasn't sure about the solution, I substituted it for x on one side of the equation and calculated the result. Then I compared the result with the other side of the equation to see if the equation balanced.
- A few times, I wrote the equation and the steps in the solution. Usually, I solved the equation mentally.

2. When you worked on your own square, what strategies did you use to put your puzzle pieces together?

- I started with the centre card that had the circle around the shape. I solved the equations on that card and searched for cards with those solutions. Then I searched for cards with equations that matched the solutions on the centre card.
- I noticed that a lot of cards had $\mathrm{x}=10$ and I didn't think that many equations would have 10 as the solution. So, I put the sides with $\mathrm{x}=10$ around the outside.

3. What strategies did your group try when putting the four square puzzles together?

- First, we figured out whose squares should be side by side.
- We solved a few questions, and then looked for matching solutions.


## Activity 2: Four Puzzles

Number of Players: team of four
Goal: to match equations and solutions to put square puzzle pieces together as a larger square

## Materials

- four sets of puzzle cards (9 clubs, 9 diamonds, 9 hearts, and 9 spades)
- pencils and paper


## How to Play

- Each player takes a different set of nine puzzle cards: spades, hearts, diamonds, or clubs.
- Each player matches equations and solutions on his or her puzzle cards to form a square consisting of three rows of three puzzle cards each (as shown below with the equation $3 x+2=5$ and the solution $x=1$ ). (The card for the centre of the square has a circle around the spade, heart, diamond, or club.)

- Use a pencil and paper for calculating, if it is helpful.
- If some players on your team finish before others, they should assist their team members.
- When the four individual puzzles are finished, work together as a team to join the four puzzles together into one large square puzzle.


## Variation

As a solitaire puzzle, one student can build each of the four squares, and then arrange them as the four parts of the puzzle. Alternatively, one student can build one puzzle of nine cards.

## Activity 2: Four Puzzles Cards (page 1)



## Activity 2：Four Puzzles Cards（page 2）

|  | ¢ 1 ＂1 n | $0 \mathrm{l}=x$  $4 x-2=34$ | $\begin{aligned} & \underset{\sim}{N} \\ & \\| \\ & \underset{X}{\prime} \end{aligned}$ |  | $0 l=x$ | ㅇ <br> 11 <br> $\times$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & - \\ & \stackrel{\prime}{\prime} \\ & \times \end{aligned}$ | $6=x$  <br> $2 x-1=41$ | $\begin{aligned} & \infty \\ & \\| \\ & \times \end{aligned}$ |  | $\text { 乙દ }=x 乙$ | $\stackrel{0}{\text { II }}$ |
|  | $\begin{aligned} & N \\ & \underset{X}{\\|} \end{aligned}$ | $\text { ル }=x$ <br> ガ $ا+9 \cdot z=x z$ <br> $x=5$ |  | $\times$ + $\sim$ $\sim$ $\stackrel{\rightharpoonup}{\sigma}$ | $L=l+x$  $x=8$ | ㅇ <br> 11 <br> $\times$ |

## Activity 2: Four Puzzles Cards (page 3)



## Activity 2: Four Puzzles Cards (page 4)

|  | ¢ $\times$ $\times$ $\times$ |  | $\begin{aligned} & 0 \\ & \\| \\ & \times \end{aligned}$ | $\begin{gathered} N \mid x \\ + \\ + \\ \\| \\ \stackrel{\rightharpoonup}{N} \end{gathered}$ | $9 \mathrm{~S}=x_{L}$ $x=1$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{s}=x$  $3 x-4=14$ | $\begin{aligned} & \stackrel{\text { ®}}{\dagger} \\ & \vdots \\ & \star \\ & \star \end{aligned}$ | $\begin{aligned} & x \\ & 11 \\ & 1 \end{aligned}$ | $l=\varepsilon-x_{t}$  $x=6$ |
|  | $\top$ <br>  <br> $\\|$ <br> $\times$ | $9=x$  $x=10$ |  | $\begin{aligned} & x \\ & 11 \\ & 0 \end{aligned}$ | $\succ l=\nabla-x \varepsilon$  $x=10$ |

## Part B: Concepts and Skills Assessment

## Administration

This assessment addresses specific expectations within two of the three overall expectations in the Grade 8 Number Sense and Numeration strand and within the two overall expectations in the Grade 8 Patterning and Algebra strand that relate to the Algebra part of the Number Sense and Algebra strand of Grade 9 Principles of Mathematics (Academic). Part B includes several styles of questions: short response, explanation, fill-in-the-blank, and multiple choice.

## Materials

| FOR THE TEACHER | FOR EACH STUDENT |  |
| :--- | :--- | :--- | OPTIONAL MATERIALS

## Introducing the Assessment

Inform students that they will be completing an assessment to help you know what they have learned about math in earlier grades. Tell students that it is important that they answer the questions as fully as possible. To communicate effectively, they can use pictures, numbers, words, and/or diagrams to represent their thinking.

## Accommodating Students with Special Needs

If individuals or groups of students have difficulty with reading, consider reading the questions aloud as they complete the assessment. If possible, a few students may benefit from working in a resource room.

If individual students have difficulty explaining their thinking in writing, consider providing scribes to record for the students, or encourage students to show and explain their thinking using concrete materials.

Some students will require additional time to complete the assessment. You may wish to note this accommodation in your anecdotal records about the student. However, there should be no reduction of the student's overall score in terms of the amount of time it takes the student to complete the assessment.

## Scoring the Assessment

Student scores may be recorded on each assessment and transferred to the Class Tracking Sheet on pages 94 and 95 , or recorded directly on the Class Tracking Sheet. Alternatively, you may record individual scores on the Individual Student Scoring Guide on pages 90 to 93 to keep a detailed record of the results for some students, and then transfer the results to the Class Tracking Sheet. The results may also be recorded electronically using the ONAP 9 CD-ROM.

While great care has been taken to consider the range of possible answers for each question, there will be times when you will need to apply your professional judgment to score an individual answer. You may use the Ontario Curriculum Correlation chart provided on pages 96 and 97 to help you determine whether the student has demonstrated the intended concept, knowledge, or skill based on the overall and specific expectations being assessed by the particular question.

At times, a student may provide an answer that you think does not completely represent his or her knowledge and skill level. You may ask probing questions to better assess the student's overall understanding.

Name:
Date: $\qquad$

## Algebra

1. Write $3 \times 3 \times 3 \times 3 \times 3$ as a power.
2. Evaluate the expression $-2[1+(-5)-(-3)+8-2]$.

Show your work.
$-2[1+(-5)-(-3)+8-2]=$
3. a) Complete the table of values for the algebraic expression $3 n-2$.

| Term number (n) | Term value |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

b) Which is the value of $4 n+3$ when $n=20$ ?

A 23
B 83
C 423
D 92

Name: $\qquad$ Date: $\qquad$
4. Jerald has some quarters and dimes in a jar.

Which algebraic equation could you use to calculate the total value, $v$, in dollars for any number of quarters and dimes, where $q$ represents the number of quarters and $d$ represents the number of dimes?
A $\quad v=0.25+0.10$
B $\quad v=25 d+10 q$
C $\quad v=0.25 q+0.10 d$
D $\quad v=25 q+10 d$
5. a) Write an algebraic expression for 14 less than 5 times a number.
b) Write an equation for this situation: Twice Gillian's age is 28 . You do not need to solve the equation.
6. a) Evaluate $3 x y+7 y$, if $x=3$ and $y=-1$.

Show your work.
$3 x y+7 y=$

Name: Date:
6. b) Evaluate $20 p-4 q+10 r$, if $p=0.3, q=0.5$, and $r=0.1$.

Show your work.
$20 p-4 q+10 r=$
c) Evaluate $4 x-6 y+7 z$, if $x=\frac{1}{2}, y=\frac{1}{3}$, and $z=1$.

Show your work.
$4 x-6 y+7 z=$

Name: Date:
7. a) Solve the equation $4 w-3=9$. Explain or show your strategy with your solution.
$4 w-3=9$
b) Jagger solved the equation $-3 x+4=-2$ and determined that $x=-2$ is the solution.

Is Jagger correct?
Explain how you know.

Name: $\qquad$ Date:
8. a) Reena created this pattern. Complete the table of values for Reena's pattern.


Figure 1


Figure 2


Figure 3

| Figure number | Number of circles |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 | 20 |
| 4 |  |
| 5 |  |

b) Write an algebraic expression for the number of circles for any figure number, $n$.
c) Selina created a pattern using triangles. Her pattern can be represented by the algebraic expression $7 n+5$, where $n$ is the figure number. She extended her pattern and wrote the equation $7 n+5=82$. Solve the equation and explain what the solution tells her about her pattern.

Show your work.

The solution to the equation is $n=$ $\qquad$ .

Explain what this tells you about her pattern.

## ONAP INDIVIDUAL STUDENT SCORING GUIDE

 GRADE 9: ALGEBRA-PART BName:
Date: $\qquad$

Overall Expectation 8m8 (Quantity Relationships):
Represent, compare, and order equivalent representations of numbers, including those involving positive exponents.

| $8 \mathrm{ml11}$ |  |  |  |
| :--- | :--- | :--- | :--- |
| 1. | $3^{5}$ <br> 1 point for the correct answer | Total for Overall Expectation 8m8 |  |
|  |  | -1 |  |

Overall Expectation 8m9 (Operational Sense):
Solve problems involving whole numbers, decimal numbers, fractions, and integers, using a variety of computational strategies.

| $\begin{aligned} & 8 \mathrm{~m} 23 \\ & 2 . \end{aligned}$ | $\begin{aligned} -2[1+(-5)-(-3)+8-2] & =-2[1-5+3+8-2] \\ & =-2[5] \\ & =-10 \end{aligned}$ <br> 2 points for the correct answer with correct steps following the order of operations, OR <br> 1 point if the answer is incorrect but demonstrates knowledge of the order of operations (e.g., the student erroneously calculates a value, but continues the solution correctly from the point that the error was introduced), OR <br> 1 point for the correct answer, but with no work shown |  |
| :---: | :---: | :---: |
|  | Total for Overall Expectation 8m9 | 2 |

Overall Expectation 8m54 (Patterns and Relationships):
Represent linear growing patterns (where the terms are whole numbers) using graphs, algebraic expressions, and equations.


| $\begin{aligned} & 8 \mathrm{~m} 58 \\ & \text { 3. b) } \end{aligned}$ | B 83 <br> 1 point for the correct answer |  |
| :---: | :---: | :---: |
|  | Total for Overall Expectation 8m54 | 3 |
| Overa <br> Mode using | Expectation 8m55 (Variables, Expressions, and Equations): <br> inear relationships graphically and algebraically, and solve and verify algebraic equations, variety of strategies, including inspection, guess and check, and using a "balance" model. |  |
| $\begin{aligned} & 8 \mathrm{~m} 59 \\ & 4 . \end{aligned}$ | $\mathbf{C} v=0.25 q+0.10 d$ <br> 1 point for the correct answer |  |
| 8 m 61 <br> 5. a) | $5 x-14$ <br> (students could use a different variable or record the expression differently, such as $x+x+x+x+x-14$ or $5 \times a-14)$ <br> 1 point for a correct expression |  |
| 8 m 61 <br> 5. b) | $2 x=28$ <br> (students could use a different variable or write the equation with values rearranged, such as $n+n=28$ or write $a=14$ ) <br> 1 point for a correct equation |  |
| $\begin{aligned} & 8 \mathrm{~m} 62 \\ & \text { 6. a) } \end{aligned}$ | $\begin{aligned} 3(3)(-1)+7(-1) & =-9-7 \\ & =-16 \end{aligned}$ <br> 2 points for the correct answer with work shown (students could determine the value of each term separately and then combine the results, e.g., $3(3)(-1)=-9,7(-1)=-7,-9+(-7)=-16)$, OR 1 point if the answer is incorrect but demonstrates understanding of some components of the question (e.g., the student erroneously calculates $3(3)(-1)=9$, but continues the solution correctly), OR <br> 1 point for the correct answer, but with no work shown |  |
| $\begin{aligned} & 8 \mathrm{~m} 62 \\ & 6 . \mathrm{b}) \end{aligned}$ | $\begin{aligned} 20(0.3)-4(0.5)+10(0.1) & =6-2+1 \\ & =5 \end{aligned}$ <br> $\mathbf{2}$ points for the correct answer with work shown (students could determine the value of each term separately and then combine the results, e.g., $20(0.3)=6,-4(0.5)=-2,10(0.1)=1$, $6-2+1=5$ ), OR <br> 1 point if the answer is incorrect but demonstrates understanding of some components of the question (e.g., the student erroneously calculates $20(0.3)=60$, or substitutes the numbers into the expression incorrectly, but continues the solution correctly), OR <br> 1 point for the correct answer, but with no work shown |  |


| 8 m 62 <br> 6. c) | $\begin{aligned} 4\left(\frac{1}{2}\right)-6\left(\frac{1}{3}\right)+7(1) & =2-2+7 \\ & =7 \end{aligned}$ <br> 2 points for the correct answer with work shown (students could determine the value of each term separately and then combine the results, e.g., $4\left(\frac{1}{2}\right)=2,-6\left(\frac{1}{3}\right)=-2,7(1)=7$, $2-2+7=7), \text { OR }$ <br> 1 point if the answer is incorrect but demonstrates understanding of some components of the question (e.g., the student erroneously calculates $6\left(\frac{1}{3}\right)=3$, or substitutes the numbers into the expression incorrectly, but continues the solution correctly), OR <br> 1 point for the correct answer, but with no work shown |  |
| :---: | :---: | :---: |
| $\begin{aligned} & 8 \mathrm{~m} 64 \\ & \text { 7. a) } \end{aligned}$ | $\begin{aligned} 4 w-3 & =9 \\ 4 w-3+3 & =9+3 \\ 4 w & =12 \\ 4 w \div 4 & =12 \div 4 \\ w & =3 \end{aligned}$ <br> Or, I thought of counters. I added 3 counters to $4 \mathrm{w}-3$ so that only 4 w would be on the left. I added 3 counters to 9 so that the sides would be balanced. That gave 4 w on the left and 12 on the right. I divided 4w by 4 to divide into 4 equal groups. I divided 12 into 4 equal groups to keep the sides of the equation balanced. That left won the left and 3 on the right, so w $=3$. <br> $\mathbf{2}$ points for the correct answer and using an appropriate strategy (e.g., balance model, counters, guess and check), OR <br> 1 point if the answer is incorrect but demonstrates understanding of a component of the question (e.g., the student selected a correct strategy to solve the equation, such as a balance model, counters, or guess and check, but came up with the incorrect answer), OR <br> 1 point for the correct answer, but with no strategy given |  |
| $\begin{aligned} & 8 \mathrm{~m} 64 \\ & \text { 7. b) } \end{aligned}$ | No. Possible explanations include: <br> Check: <br> Left side Right side $\begin{aligned} & -3 x+4 \\ & =-3(-2)+4 \\ & =6+4 \\ & =10 \end{aligned}$ $-2$ <br> When I substitute -2 into the equation, the value of the left side does not equal the value of the right side. <br> Solve: $\begin{aligned} -3 x+4 & =-2 \\ -3 x+4-4 & =-2-4 \\ -3 x & =-6 \\ x & =2 \end{aligned}$ <br> So $x$ does not equal -2 . |  |


| 7. b) (Cont'd) | $\mathbf{2}$ points for the correct answer supported by an appropriate explanation, OR <br> 1 point if the answer is incorrect but demonstrates understanding of the question (e.g., the student selected a correct strategy to explain the answer, such as a balance model, counters, or guess and check, but came up with the incorrect answer), OR <br> 1 point for the correct answer, but with no explanation given |  |
| :---: | :---: | :---: |
| $\begin{aligned} & 8 \mathrm{~m} 63 \\ & 8 . \mathrm{a}) \end{aligned}$ | Figure number Number of circles <br> 1 8 <br> 2 14 <br> 3 20 <br> 4 26 <br> 5 32 <br> $\mathbf{2}$ points for 4 correct term values, OR <br> 1 point for at least 2 correct term values |  |
| $\begin{aligned} & 8 \mathrm{~m} 63 \\ & 8 . \mathrm{b}) \end{aligned}$ | $6 n+2$ <br> (students could use a different variable or record the expression differently, (such as $2+6 n \text { or } 2+n+n+n+n+n+n)$ <br> 1 point for a correct expression |  |
| $\begin{aligned} & \text { 8m63 } \\ & 8 . \mathrm{c}) \end{aligned}$ | $\begin{aligned} 7 n+5 & =82 \\ 7 n+5-5 & =82-5 \\ 7 n & =77 \\ 7 n \div 7 & =77 \div 7 \\ n & =11 \end{aligned}$ <br> The number of the figure with 82 triangles is 11 . <br> $\mathbf{2}$ points for the correct solution and correct explanation about what it represents, using an appropriate strategy, OR <br> $\mathbf{1}$ point if the solution is incorrect but provides a reasonable explanation about what the solution represents (e.g., the solution tells her the number of the figure that contains 82 triangles), OR <br> $\mathbf{1}$ point for the correct solution, but an incorrect explanation of what the solution represents |  |
|  | Total for Overall Expectation 8m55 | 18 |

## ONAP GRADE 9: ALGEBRA

Date:
Grade: $\qquad$ School: $\qquad$


NOTE: IEP/ELL means Individualized Education Plan/English Language Learner.

## CLASS TRACKING SHEET-PART B

Board: $\qquad$ Teacher Name: $\qquad$

8m55 (Variables, Expressions, and Equations)
Model linear relationships graphically and algebraically, and solve and verify algebraic equations, using a variety of strategies, including inspection, guess and check, and using a "balance" model.


## ONTARIO CURRICULUM CORRELATION TO ONAP ALGEBRA 9-PART B <br> NOTE: This correlation is to the Grade 8 Ontario Curriculum Expectations.

|  | ONTARIO CURRICULUM CORRELATION TO ONAP ALGEBRA 9-PART B NOTE: This correlation is to the Grade 8 Ontario Curriculum Expectations. |
| :---: | :---: |
| Overall Expectation 8m8 (Quantity Relationships): <br> Represent, compare, and order equivalent representations of numbers, including those involving positive exponents. |  |
| Question Number | Specific Expectation |
| 1. | 8m11: express repeated multiplication using exponential notation (e.g., $2 \times 2 \times 2 \times 2=2^{4}$ ) |
| Overall Expectation 8m9 (Operational Sense): <br> Solve problems involving whole numbers, decimal numbers, fractions, and integers, using a variety of computational strategies. |  |
| Question Number | Specific Expectation |
| 2. | 8m23: evaluate expressions that involve integers, including expressions that contain brackets and exponents, using order of operations |
| Overall Expectation 8 m 54 (Patterns and Relationships): <br> Represent linear growing patterns (where the terms are whole numbers) using graphs, algebraic expressions, and equations. |  |
| Question Number | Specific Expectation |
| 3. a)-b) | 8m58: determine a term, given its term number, in a linear pattern that is represented by a graph or an algebraic equation |
| Overall Expectation 8m55 (Variables, Expressions, and Equations): <br> Model linear relationships graphically and algebraically, and solve and verify algebraic equations, using a variety of strategies, including inspection, guess and check, and using a "balance" model. |  |
| Question Number | Specific Expectation |
| 4. | 8m59: describe different ways in which algebra can be used in real-life situations (e.g., the value of $\$ 5$ bills and toonies placed in an envelope for fund raising can be represented by the equation $v=5 f+2 t)$ |
| 5. a)-b) | 8m61: translate statements describing mathematical relationships into algebraic expressions and equations (e.g., for a collection of triangles, the total number of sides is equal to three times the number of triangles or $s=3 n$ ) |


| Question <br> Number | Specific Expectation |
| :---: | :--- |
| 6. a)-c) | 8m62: evaluate algebraic expressions with up to three terms, by substituting fractions, decimals, and <br> integers for the variables (e.g., evaluate $3 x+4 y=2 z$, where $x=\frac{1}{2}, y=0.6$, and $\left.z=-1\right)$ |
| 7. a)-b) | 8m64: solve and verify linear equations involving a one-variable term and having solutions that are <br> integers, by using inspection, guess and check, and a "balance" model |
| 8. a)-c) | 8m63: make connections between solving equations and determining the term number in a pattern, <br> using the general term (e.g., for the pattern with the general term $2 n+1$, solving the equation <br> $2 n+1=17$ tells you the term number when the term is 17) |

## Next Steps for Algebra

## Instructional Next Steps for Overall Expectations

After summarizing individual and class performance on the overall expectations, you may find that there are Grade 8 concepts that require remediation on either an individual or a group basis. References have been made to the following resources to assist you in preparing tasks for individuals or for small groups of students.

- Nelson Mathematics 7: Student Text, Student Success Workbook
- Nelson Mathematics 8: Student Text, Student Success Workbook
- PRIME (Professional Resources and Instruction for Mathematics Educators)
- BIG IDEAS from Dr. Small: Creating a Comfort Zone for Teaching Mathematics, Grades 4-8, by Marian Small
- Making Math Meaningful to Canadian Students, K-8, by Marian Small
- Good Questions: Great Ways to Differentiate Mathematics Instruction, by Marian Small
- More Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction, by Marian Small and Amy Lin
- Math on Call: A Mathematics Handbook
- Algebra to Go: A Mathematics Handbook
- MATH GAINS: TIPS4RM (Targeted Implementation and Planning Supports for Revised Mathematics), Grade 7 and Grade 8, http://www.edugains.ca/newsite/math/ tips4rm.htm
- CLIPS (Critical Learning Instructional Paths Supports), http://www.oame.on.ca/CLIPS/


## Overall Expectation 8m9 (Operational Sense)

Solve problems involving whole numbers, decimal numbers, fractions, and integers, using a variety of computational strategies.

## Background

This overall expectation is about facility with number operations. Operational sense also indicates an understanding of the relationships between and among the operations of addition, subtraction, multiplication, and division, as well as knowledge of the conventions of notation and the order of operations. A sound knowledge and understanding of integer arithmetic is required if students are to be successful when adding and subtracting like terms to simplify polynomials in Grade 9 and in higher grades.

In Grade 7, students learned to order, add, and subtract integers. In Grade 8, students learned to multiply and divide integers. They used concrete models, such as coloured counters, integer tiles, and number lines, to represent positive and negative integers. They studied various strategies to perform the operations of addition, subtraction, multiplication, and division with integers. Students also learned to apply the order of operations to evaluate numerical expressions containing integers and to evaluate algebraic expressions for integer values of the variables.

## Strategies

Task: Evaluate numerical and algebraic expressions that involve operations with integers, using the rules for the order of operations.

Student Misconceptions: Students may have difficulty adding and subtracting integers. They may erroneously apply the rules for multiplying and dividing integers to the operations of addition and subtraction. For example, a student may say that $(-3)+(-4)=7$ because "two negatives make a positive." This may be compounded by a misconception that when you add two numbers, the result is a greater number, and when you subtract one number from another number, the result is less than the first number. Both of these concepts are true for the numbers students worked with prior to being introduced to integers.

Students may also have difficulty with integer notation; that is, when to use -2 or $(-2)$ and when -2 is sufficient. Some students are confused by square or nested brackets within an expression and may misuse or overlook these. For example, a student may erroneously interpret $-2[1+(-5)]$ as $-2 \times 1+(-5)$.

Some students have difficulty understanding that $-2^{2}$ and $(-2)^{2}$ result in two different values. They may erroneously interpret that the base of both powers is -2 when, in fact, this is only true for $(-2)^{2}$.

Targeting the Misconceptions: Use concrete models, such as a number line and/or integer tiles, to allow students to develop a strong understanding of the meaning of operations as they apply to integers. For example, encourage students to think of $(-3)+(-4)$ as " $(-3)$ and another $(-4)$ " and to describe $(-2)-(-5)$ as "the distance from ( -5 ) to ( -2 ) on a number line." As a check for understanding, always ask students to explain how they got their answers.

Discourage the use of superscript +'s and -'s to denote integer signs, since these are not as clear as brackets when handwritten and may be confused with operation signs. Calculators can force students to make the distinction between integer and operation signs, since there are different keys for ( - ) and - . Talk about why there is a need for square or nested brackets-in addition to round brackets-in some expressions that contain integers. Provide a great deal of practice with writing and evaluating expressions that include both integer round brackets and square or nested brackets.

When working with powers, remind students that the base of a power is only negative when it is enclosed in parentheses.
$(-2)^{2}=(-2)(-2)=4$

$$
-2^{2}=-(2)(2)=-4
$$

| Instructional strategy | Resources to help with instruction and practice |
| :---: | :---: |
| Adding and subtracting integers using coloured counters and the zero principle | - Nelson Mathematics 7, Lesson 6.3; Chapter 6 Mid-Chapter Review; Lesson 6.6; Chapter 6 Chapter Review <br> - Nelson Mathematics 8, Lesson 6.2; Chapter 6 Mid-Chapter Review <br> - BIG IDEAS from Dr. Small, Grades 4-8, pp. 86-89 <br> - Making Math Meaningful to Canadian Students, K-8, pp. 270-274 <br> - TIPS4RM, Grade 7, Unit 2, Day 8: The Zero Principle; Day 10: Add Some Colour; Day 14: What's the Difference? |
| Adding and subtracting integers using directed distances on a number line | - Nelson Mathematics 7, Lesson 6.4; Chapter 6 Mid-Chapter Review; Lesson 6.7; Chapter 6 Chapter Review <br> - Nelson Mathematics 8, Lesson 6.2; Chapter 6 Mid-Chapter Review <br> - PRIME, Number and Operations, Background and Strategies, pp. 148-149 <br> - BIG IDEAS from Dr. Small, Grades 4-8, pp. 86-89 <br> - Making Math Meaningful to Canadian Students, K-8, pp. 270, 274 <br> - TIPS4RM, Grade 7, Unit 2, Day 11: What's Right About Adding and What's Left to Count? Day 12: Adding On! |
| Using rules to add and subtract integers | - Nelson Mathematics 7, Lesson 6.5 <br> - Nelson Mathematics 8, Chapter 6 Mid-Chapter Review <br> - BIG IDEAS from Dr. Small, Grades 4-8, p. 89 <br> - Making Math Meaningful to Canadian Students, K-8, p. 275 <br> - TIPS4RM, Grade 7, Unit 2, Day 15: Integer Fun |
| Evaluating expressions using the order of operations | - Nelson Mathematics 8, Lesson 6.7; Chapter 6 Chapter Review <br> - Making Math Meaningful to Canadian Students, K-8, p. 279 |

## Overall Expectation 8m54 (Patterns and Relationships)

Represent linear growing patterns (where the terms are whole numbers) using graphs, algebraic expressions, and equations.

## Background

This overall expectation is about multiple representations for linear pattern relationships. Linear growing patterns may be described using words or pictures. The relationship between the term number and term value for a pattern may be shown as a written rule, in a table, in a graph, or as an equation.

In Grades 7 and 8 , students learned to use numbers, graphs, and variables to represent pattern relationships. They used pattern blocks, counters, and toothpicks to create growing patterns, and then they described their patterns using words, tables of values, and scatter plots. They studied various strategies to create algebraic expressions to describe patterns. Students also learned to evaluate algebraic expressions representing patterns.

## Strategies

Task: Determine a term value, given its term number, in a linear pattern that is represented by a graph or algebraic equation.

Student Misconceptions: Some students have difficulty understanding how letters are used to represent variables. They interpret $3 n$ as a number in the 30 s with ones-digit $n$, instead of the product $3 \times n$. Other students feel compelled to "simplify" algebraic expressions to a single term. For example, they erroneously simplify $3 n-2$ to $1 n$ and $4 n+3$ to $7 n$ before proceeding to the evaluation stage. Students may also make order of operation errors; that is, they may perform the addition or subtraction before the multiplication.

Targeting the Misconceptions: Ask students to translate the algebraic expression into words before evaluating it. Emphasize that in algebraic expressions, terms such as $3 n$ refer to products and are read "three times a number." In addition, provide students with practice writing algebraic expressions to describe patterns and other situations that are described in words. Encourage the use of brackets around the number to be substituted when evaluating an expression for a given value of the variable. For example, to evaluate $4 n+3$ when $n=20$, write $4(20)+3$.

| Instructional strategy | Resources to help with instruction and practice |
| :---: | :---: |
| Creating and representing pattern rules | - Nelson Mathematics 7, Lesson 4.3; Chapter 4 Mid-Chapter Review <br> - Nelson Mathematics 8, Lesson 4.2; Chapter 4 Mid-Chapter Review; Lesson 4.5; Chapter 4 Chapter Review <br> - PRIME, Patterns and Algebra, Background and Strategies, pp. 61-62 <br> - Making Math Meaningful to Canadian Students, K-8, pp. 584-585 <br> - Good Questions: Great Ways to Differentiate Mathematics Instruction, pp. 131-132 <br> - More Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction, pp. 24-25 <br> - TIPS4RM, Grade 7, Unit 2, Day 1: Toothpick Patterns; Day 2: Patterns with Tiles; Day 3: Pattern Practice <br> - CLIPS, Representations of Linear Growing Patterns: Simple Linear Growing Patterns, Robot Transformer; Comparing Families of Linear Growing Patterns, Comparing Pattern Rules |
| Translating patterns and statements into algebraic expressions | - Nelson Mathematics 7, Lesson 8.3; Chapter 8 Mid-Chapter Review <br> - Nelson Mathematics 8, Lessons 4.3, 8.3; Chapter 8 Mid-Chapter Review <br> - PRIME, Patterns and Algebra, Background and Strategies, pp. 61-62 <br> - Making Math Meaningful to Canadian Students, K-8, pp. 582-583 <br> - More Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction, pp. 19-20, 46-47, 50-51 <br> - Math on Call, 216 <br> - TIPS4RM, Grade 7, Unit 2, Day 4: Pattern Exchange <br> - TIPS4RM, Grade 7, Unit 5, Day 1: Using Variables in Expressions <br> - TIPS4RM, Grade 8, Unit 2, Day 3: Finding the nth Term; Day 4: Exploring Patterns <br> - CLIPS, Representations of Linear Growing Patterns: Linear Growing Patterns, Rocket Rules |
| Matching algebraic expressions with statements and patterns | - Nelson Mathematics 7, Lesson 8.2 <br> - Nelson Mathematics 8, Lesson 4.2; Chapter 4 Mid-Chapter Review |
| Evaluating algebraic expressions | - Nelson Mathematics 7, Lesson 8.3; Chapter 8 Mid-Chapter Review <br> - Nelson Mathematics 8, Lesson 8.3; Chapter 8 Mid-Chapter Review <br> - More Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction, 21-22, 25-27, 46-47 <br> - TIPS4RM, Grade 7, Unit 5, Day 3: Evaluating Algebraic Expressions with Substitutions |

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Overall Expectation 8m55 (Variables, Expressions, and Equations)
Model linear relationships graphically and algebraically, and solve and verify algebraic
equations, using a variety of strategies, including inspection, guess and check, and
using a "balance" model.
```


## Background

This overall expectation is about writing equations to express linear patterns and relationships. It is also about solving equations in order to answer specific questions about patterns or relationships.

In Grade 7, students learned to write equations and solve them by inspection and by systematic guessing and checking. In Grade 8, they learned how to use a graph to solve an equation, and also learned how to solve equations using a balancing strategy. They used equations to solve pattern problems and other problems with familiar real-world contexts.

## Strategies

Task: Write equations to represent linear relationships and solve and verify linear equations.

Student Misconceptions: Students may have difficulty using variables correctly in expressions to represent even familiar relationships. For example, to represent that there are 25 cents in a quarter, they may write $q=25 c$ instead of $c=25 q$, where $q$ represents the number of quarters and $c$ represents the number of cents.

Targeting the Misconceptions: Have manipulatives, such as plastic coins, counters, and toothpicks, available for students to use when writing and solving equations. Encourage students to try out their equations by substituting known values, such as values from a table that describes the pattern they are working with, or values such as $c=50$ when $q=2$ in the quarters and cents relationship.

Ask students to write several different numerical expressions for a given number. For example, if the number is 7 , they could write $1+6,4+3,10-3,1+2+4$, or $2 \times 3+1$. Then ask them to use the variable $x$ in algebraic expressions so that the value of the expression is 8 . An example is when $x=2$, they can write expressions such as $4 x, 5 x-2, x+6,2 x+2+x$. Finally, have students build equations by equating these expressions two at a time. They can then verify by substitution that left side $=$ right side $(=8)$, when $x=2$.

| Instructional strategy | Resources to help with instruction and practice |
| :--- | :--- |
|  | - PRIME, Patterns and Algebra, Background and Strategies, p. 68 <br> - BIG IDEAS from Dr. Small, Grades 4-8, p. 10 |
| Exploring the concept <br> of equality by creating <br> equivalent equations |  |
| - Making Math Meaningful to Canadian Students, K-8, p. 585 |  |
| - Good Questions: Great Ways to Differentiate Mathematics |  |
| Instruction, p. 135 |  |
| - More Good Questions: Great Ways to Differentiate Secondary |  |
| Mathematics Instruction, pp. 21-24, 28-29 |  |



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